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The Special Volume on the Occasion of 50th Anniversary of the Department of Statistics

JUJSS

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The special issue of Jahangirnagar University Journal of Statistical Studies (JUJSS) is to celebrate the 50-year founding anniversary of the Department of Statistics which is the first four founding departments of Jahangirnagar University. This milestone anniversary also coincides with two other memorable anniversaries: the 100-year birth celebration of the father of the nation Bangabandhu Sheikh Mujibur Rahman and the 50^{th} year of independence of Bangladesh. The department of Statistics and JUJSS sincerely acknowledge the funding support from the Jahangirnagar University and the Professional Master Program in Applied Statistics and Data Sciences (PMASDS-JU) program. Department and the JUJSS also sincerely acknowledge the dedicated effort of the editorial board members, reviewers, and contributors.

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The Jahangirnagar University Journal of Statistical Studies (JUJSS), a publication of the Department of Statistics, publishes original contributions based on the research study in statistical theory in the widest sense of the term and its applications in various fields. The contributors of this Journal are requested to follow the notes mentioned below:

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Address for Communication:

Professor Dr. Mohammad Alamgir Kabir

Executive Editor, JUJSS

Department of Statistics, Jahangirnagar University
Savar, Dhaka-1342, Bangladesh
Telephone: +8801711979979 (WhatsApp)
Fax: +88-02-7791052
Email:editor.jujss@juniv.edu; CC to: salam@juniv.edu, alamgirfa_juniv@yahoo.com

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Editorial Notes

The Department of Statistics of Jahangirnagar University is one of the four departments with which university started its journey in 1971. Its goal is always to build the department as a center of excellence, providing the highest quality education and leading-edge research. To enhance scientific research, it has been publishing Jahangirnagar University Journal of Statistical Studies (JUJSS) since 1981. Statistics as a discipline has been through a big transformation over the last fifty years. The statistics community in recent years has been observed significant advancement in the area of statistical computing, inferences, data mining, artificial intelligence, and big data that together led to the emerging area of data science. The field of statistics is changing continuously in response to the remarkable increase in demand for statistical thinking and methodology in scientific research. The universal use of statistical ideas has required us to adapt both our teaching and research quickly.

The Department of Statistics has decided to publish a special volume (volume 36) of the journal to commemorate the glorious 50 years of its journey. This glorious moment of the department as well as the university is also coinciding with the 50th anniversary of the independence of the nation, Bangladesh in 2021. It is also mentioned that the country is also celebrating the 100th birth anniversary of the father of the nation Bangabandhu Sheikh Mujibur Rahman from 2020. How often do such coincidences occur in almost the same time? The likelihood certainly is very small for any nation, and most likely this has uniquely happened in our case. We are proud to celebrate these occasions, and what could be a better way to celebrate it than bringing a special issue for the readers of the JUJSS?

From its inception, the JUJSS has been a proud host of many articles that reshaped statistical methodology and application in the emerging areas of statistics. While it is impossible to cover such a vast area in one special issue, we have assembled an outstanding group of nationally and internationally renowned researchers mostly from former students of the department of statistics to contribute to this special issue to celebrate the 50 years founding anniversary of the department. The main objective of this special volume is to make a bridge among the JU researchers from home and abroad. Under this backdrop, this volume contains invited articles from many scholars across the globe. We are thankful to all the contributors for their timely and significant contributions. This special issue brings together a wonderful collection of fourteen articles that represent many brunches of statistics and covers wide areas of classical and modern theoretical and applied statistics. We anticipate that this special volume will be interesting and useful to the other researchers to understand and foster further research in several branches of statistics. We are thankful to all the contributors for submitting their acclaimed work to JUJSS, while they certainly had the option to choose other journals.

Finally, we would like to express our gratitude to all the reviewers and the editorial board members of the journal for giving invaluable comments, enormous voluntary contributions and their valuable time for the successful release of this volume. Further, it would have not been possible publish it without the outstanding effort of the total JUJSS team. The Department of Statistics and JUJSS sincerely acknowledge the funding support from the Jahangirnagar University and the department run Professional Master Program in Applied Statistics and Data Sciences (PMASDS-JU).

On behalf of JUJSS Editorial Board
Professor Dr. Mohammad Alamgir Kabir
Executive Editor, JUJSS

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The Robust Wald Test for Testing a Subset of Regression Parameters of a Multiple Regression Model with Apriori Information on Another Subset

Shahjahan Khan*
School of Mathematics, Physics and Computing, School of Science and Techonology, Asian University of Bangladesh, Dhak 1341, Bangladesh and University of Southern Queensland, Toowoomba, Qld 4350, Australia.

Rossita M Yunus
Institute of Mathematical Sciences, Faculty of Sciences, University of Malaya, 50603 Kuala Lumpur, Malaysia.

Abstract
The classical and M-estimator-based robust Wald tests are introduced to simultaneously test an arbitrary subset of coefficients of a multiple regression model when the remaining coefficients are either (i) unspecified, (ii) specified with certainty or (iii) suspected with uncertainty. Under the three scenarios the classical and robust Wald test statistics for (i) unrestricted (UT), (ii) restricted (RT) and (iii) pre-test (PTT) tests are defined. The aims of the paper are to (i) define the classical and robust Wald UT, RT and PTT statistics, (ii) find the asymptotic distribution of the test statistics (iii) determine the power function of the tests and (iv) compare the performance of the robust Wald UT, RT and PTT to their classical counterparts for large data. A Monte Carlo simulation study is conducted to obtainand compare the empirical power of the tests. The simulation study shows a domination of the PTT over the UT and RT when the suspected values are close to the true values and the robust Wald test is better than the its classical counterpart in terms of size and power under a slight departure from normality assumption. An example with Olympic athlete data is provided for illustration of the proposed method.

Keywords: M-estimator, robust test, nonparametric test, Wald test, regression model, asymptotic power, Monte Carlosimulation and contiguity.
Mathematics Subject Classification: 62E20, 62G35, 62F03, 62J05

1. Introduction

In many disciplines, results from previous studies or knowledge of experts in the field may pro-vide valuable prior information on the value of the underlying parameters of a multiple linear regression model. In general, inclusion of any trustworthy prior information in the estimation of parameter and test of hypotheses may improve the quality of statistical inference. Although the prior information usually comes from trusted sources, there is always an element of uncer-tainty in such information. The idea of the removal of the uncertainty in the non-sample prior information (NSPI) through a preliminary testing (pre-test) has drawn an increasing attention in the statistics literature since Bancroft (1944). However, almost all the initial studies in this area were focusing on improving estimation of parameters rather than hypothesis test. Ahmed and Saleh (1989), Akritas et al. (1984), Khan and Saleh (2001), Khan (2000,

* Corresponding author: shahjahan.Khan@usq.edu.au

2003, 2005), and Saleh (2006), to name a few, are among contributors to the study on developing and improving estimators of various kind of statistical models through preliminary testing. Despite the plethora of studies into the effects of pre-test on estimation, there are only few studies that have involved the

In statistics, it is a general interest to increase the power of any test, and it is more so for the testing after pre-test problem. Statistician shave studied the effect of pre-testing on the final test for several models including the analysis of variance (Paull 1950, Bechhofer 1951, Bozivich et al.1956, Mead et al.1975, among others), one sample and two sample problems (Tamura 1965), simple regression, multiple regression, multivariate regression and parallelism models (Saleh and Sen 1982,1983, Lambert et al. 1985, Yunus and Khan 2010, 2011a, b). In the literature, the statistical tests that were used in these models are based on the rank tests (Tamura 1965, Saleh and Sen 1982, 1983) and the robust score tests (Yunus and Khan 2010, 2011 a, b) for large sample size and the t-test (Khan and Pratikno 2013) for small sample size. The feature that distinguishes this paper from the previous works in the context of multiple regression model is the introduction of classical and robust Wald test in the preliminary testing frame work. Furthermore, it defines and investigates the size and power of classical and robust Wald tests in the context of testing an arbitrary subset of regression parameters where prior information on the other subset is available. It also provides an illustrative example of the proposed test using Olympic athlete data.
Consider the following multiple regression model

$$Y_n = X_n \beta + e_n, \tag{1.1}$$

where $Y_n = (Y_1, Y_2, ..., Y_n)'$ is a vector of *n* realizations of an observable response variable, $\mathbf{X}_n$ is a known design matrix of order $n \times p$, $\beta = (\beta_1, \beta_2, ..., \beta_p)'$ is a *p*-dimensional row vector of unknown regression parameters, and $\mathbf{e}_n = (e_1, e_2, \ldots, e_n)'$ is a vector of *n* independent and identically distributed errors with a distribution function *F*.

To formulate the testing of an arbitrary subset of the regression parameter vector, let's partition the *p*-dimensional parameter vector β' as $\beta' = (\beta_1', \beta_2')$ with $\beta_1' = (\beta_1, ..., \beta_r)$ and $\beta_2' = (\beta_{r+1}, ..., \beta_p)$ two *r* and *t* dimensional row vectors such that $r + t = p$. Then, partition X_n' as (X_{n1}', X_{n2}'), where $\mathbf{X}_{n1}$ is a known design matrix of order $n \times r$ and $\mathbf{X}_{n2}$ is another known design matrix of order $n \times t$.

Consider testing the parameters vector $\boldsymbol{\beta}_1$ specified at $\boldsymbol{\beta}_{01}$ when there is uncertain NSPI on $\boldsymbol{\beta}_2$, namely values (i) unspecified (ii) specified and (iii) suspected but not sure. For case (i), we want to test $H_0^*: \beta_1 = \beta_{01}$ against $H_A^*: \beta_1 \neq \beta_{01}$ with $\boldsymbol{\beta}_2$ is treated as a nuisance parameter.

This test is called the unrestricted test (UT). For case (ii), the test for testing $H_A^*: \beta_1 = \beta_{01}$ against $H_A^*: \beta_1 \neq \beta_{01}$ when $\beta_2 = \beta_{02}$, is called the restricted text (RT). For case (iii), testing $H_0^{(1)}: \beta_2 = \beta_{02}$, is recommended to remove the uncertainty of the suspicious values of $\beta_2 = \beta_{02}$ before testing on β_1. The test on $H_0^{(1)}: \beta_2 = \beta_{02}$ against $H_A^{(1)}: \beta_2 \neq \beta_{02}$ is known as a pre-test (PT). If the null hypothesis is this pre-test rejected, the UT is appropriate to test H_0^*, otherwise the RT is used to test H_0^*. The final test for testing for testing H_0^*, following a pre-test on $H_0^{(1)}$, is termed as the pre-test test (PTT). The objective of this study is to determine which of the classical and robust Wald UT, RT and PTT is better in terms of the test power criterion.

The next section briefly reviews the classical and robust Wald tests. Section 3 derives the asymptotic distribution of robust M-estimator sunder the null hypothesis. The robust Wald test statistic for the PT to test $H_0^{(1)}$, and the UT, RT and PTT to finally test H_0^* are introduced in Section 4. In the same section, we also provide the classical counterpart for robust Wald test. In Section 5, the Monte Carlo simulation for the comparison of the power of the tests are performed and the results are presented graphically. An illustrative example on the application of the method is provided in Section 6 with Olympic athlete data. Some concluding remarks are included in Section 7. Interested readers may refer to Appendix B and C for the asymptotic distribution under a sequence of local alternatives required for the derivation of the asymptotic power functions of the proposed robust Wald UT, RT and PTT. R-codes for the simulation study is available upon request.

2. Classical and Robust Wald Tests

Wald test is one of the classical tests that is widely used in statistics and econometrics. Originally proposed by Wald (1943), the test has been used to test parameters of linear models by many authors including Engle (1984). When F represents a multivariate normal distribution with mean vector $\mathbf{0}$ and covariance matrix $\sigma_0^2 \boldsymbol{I}_n$, $\sigma_0^2>0$, the test statistic for the classical Wald test for testing the simple null hypothesis $H_0 = \beta = \beta_0$ against $H_A: \beta \neq \beta_0$ is defined as

$$CW_n = \left(\breve{\beta} - \beta_0\right)' \sum_n^{-1} \left(\breve{\beta} - \beta_0\right), \tag{2.2}$$

where $\breve{\beta} = (X_n' X_n)^{-1} X_n' Y_n$ is the maximum likelihood estimator of β, and $\hat{\sum_n} = \frac{t}{n} \hat{\sigma}_0^2 \left(X_n' X_n\right)^{-1}$ where $\hat{\sigma}_0^2 = \sum \left(Y_i - X_i'\beta\right)^{2/n}$, in which Y_i is the response variable ion individual i and vector X_i is the ith row of the design matrix X_n, for i = 1, 2, ...,n.

The reason for preferring the Wald test over other alternative tests is its simple formulation. The Wald test is defined using the estimated coefficient and the variance of the estimator. The Wald test is easier to implement than its competitors such as the score test (Rao 1948), as it does not require the computation of the score function and the inversion of the information matrix (Carolan and Rayner 2000). The Wald and score tests for simple null hypothesis are asymptotically equivalent for large sample size (cf. Atkinson and Lawrance 1989, Rayner and Best1989, p.42). The performance of the tests can vary significantly for small samples. Un-like the likelihood ratio test (Neyman and Pearson 1928), the Wald test does not require the computation of the maximum likelihood function of the parameters both under the null and alternative hypotheses (Sen et al. 2010, pp.77). The likelihood ratio test has widely been used to test for the significance of a subset of parameters of a multiple regression model. According to Engle (1984, pp.792), the Wald (specifically the Wald UT) and likelihood ratio tests are asymptotically equivalent but the Wald test is computationally easier.

Most real-world data sets contain outliers, and thus do not follow the commonly assumed normal distribution. The classical Wald test defined in equation (2.2), however, is highly sen-sitive to model mis-specification and presence of outlying observations. Several versions of robust Wald test for linear model appear in Carroll and Ruppert (1988, pp. 214), Heritier and Ronchetti (1994), Jurečkováand Sen (1996,pp.419) and Silvapulle (1992). Recently, Basuet al. (2018) used minimum density power divergence estimator to define Wald test for multiple linear regression model. Using the same estimator Basuetal. (2017) defined Wald type test for the logistic regression model.

In this paper, we use M-estimator to define robust Wald test. This method is used to define alternative Wald tests, namely the UT, RT and PTT when uncertain non-sample prior information is available, and compared with their classical counterparts. A robust version of Wald test that is given in Jurečková and Sen (1996, pp.419) is defined below

$$RW_n = \frac{\tilde{\gamma}^2}{\tilde{\sigma}^2}\left(\tilde{\beta}-\beta_0\right)'\left(X_n'X_n\right)\left(\tilde{\beta}-\beta_0\right), \tag{2.3}$$

where $\tilde{\gamma}=\sum\left({}_nS_n\right)^{-1\psi'}\left(\frac{Y_i-X_i'\tilde{\beta}}{S_n}\right)$ and $\tilde{\sigma}^2=n^{-1}\sum\psi^2\left(\frac{Y_i-X_i'\tilde{\beta}}{S_n}\right)$ are respectively estimates of $\gamma=\frac{1}{s}\int_{-\infty}^{\infty}\psi'(u/s)\,dF(u)$ and $\sigma^2=\int_{-\infty}^{\infty}\psi^2(u/s)\,dF(u)<\infty$. Note here $\tilde{\beta}$ is a robust M-estimator of β, studentized by S_n, (cf. Jurečková and Sen,1996, pp. 216), and is the solution to

$M_n(\beta) = \sum X_i'\psi\left(\frac{Y_i - X_i'\beta}{S_n}\right) = 0$, where Y_i is the responses variable on individual i, vector X_i is the ith row of the designed matrix X_n, the function ψ is a nondecreasing and skew symmetric score function, in the sense of Huber (1981). Here S_n is a scale statistic for estimating s = s(F), the scale parameter of distribution F. S_n is the scaled median absolute deviation (MAD) of $\left(Y_i - X_i'\tilde{\beta}\right)$. The classical Wald test is a special case of the robust Wald test. In this case $s = \sigma_0^2$ and taking $\psi(u/s) = u/s$, where $u = Y - X'\beta$, gives $\gamma = 1$ and $\sigma^2 = \sigma_0^2$.

3. Asymptotic Distribution of Robust M-estimator

To find the power function of the tests defined in Section 4, the following asymptotic distributions of the robust M-estimators are required. According to Jurečková and Sen (1996, pp.216), if $\sigma^2 = \int_{-\infty}^{\infty} \psi^2(u/s)dF(u) < \infty$, $\gamma = \frac{1}{s}\int_{-\infty}^{\infty} \psi'(u/s)dF(u)$ then

$$\sqrt{n}\left(\tilde{\beta} - \beta\right) \to N\left(0, \frac{\sigma^2}{\gamma^2}Q^{-1}\right), \tag{3.4}$$

where $Q = \lim_{n\to\infty}\frac{1}{n}Q_n$ with $Q_n = X_n'X_n$.

To facilitate the derivation of the joint distribution, consider the following matrix partitioning:

$$Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} = \lim_{n\to\infty}\frac{1}{n}Q_n = \lim_{n\to\infty}\frac{1}{n}\begin{pmatrix} Q_{n11} & Q_{n12} \\ Q_{n21} & Q_{n22} \end{pmatrix}, \tag{3.5}$$

where $Q_{njk} = X_{nj}'X_{nk}$ for j, k = 1, 2. The following theorem provides the asymptotic joint distributions of the robust M-estimators under the null hypothesis

Theorem 3.1 Under $H_0: \beta_1 = \beta_{01}, \beta_2 = \beta_{02}$, asymptotically,

(i)

$$\sqrt{n}\begin{pmatrix} \tilde{\beta}_1 - & \beta_{01} \\ \tilde{\beta}_2 - & \beta_{02} \end{pmatrix} \xrightarrow{d} N_p\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \frac{\sigma^2}{\gamma^2}\begin{pmatrix} Q_1^{*-1} & Q_{12}^* \\ Q_{21}^* & Q_2^{*-1} \end{pmatrix}\right], \tag{3.6}$$

(ii)

$$\sqrt{n}\begin{pmatrix} \tilde{\beta}_1 - & \beta_{01} \\ \tilde{\beta}_2 - & \beta_{02} \end{pmatrix} \xrightarrow{d} N_p\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \frac{\sigma^2}{\gamma^2}\begin{pmatrix} Q_{11}^{*-1} & 0 \\ 0 & Q_2^{*-1} \end{pmatrix}\right], \tag{3.7}$$

where $\tilde{\beta}_1$ is the first r rows of $\tilde{\beta}_1$ $\tilde{\beta}_2$ is the last t rows of $\tilde{\beta}$ and $\tilde{\beta}_1$ is the restricted robust M-estimator of β_1, the solution of $M_{n1}(\beta_1, \beta_{02}) = 0$, where

$$M_{nj}(a,b) = \sum_{i=1}^{n} X'_{ji}\psi\left(\frac{Y_i - X'_{1i}a - X'_{2i}b}{S_n}\right) \tag{3.8}$$

for j = 1, 2 with Sn is a scale statistic of $s, a \in \Re_r$ and $b \in \Re_t$, Note here $Q_{12}^* = -Q_{11}^{-1}Q_{12}Q_2^{*-1}, Q_{21}^* = -Q_{22}^{-1}Q_{21}^{-1}Q_1^{*-1}, Q_1^* = Q_{11} - Q_{12}Q_{22}^{-1}$ and $Q_2^* = Q_{22} - Q_{21}Q_{11}^{-1}Q_{12}$.

The proof of Theorem 3.1 is given in appendix A. The asymptotic distributions of the robust M-estimators given in this section are used to obtain the power functions of robust Wald UT RT, PT and PTT defined in the next section.

4. The Classical and Robust Wald UT, RT, PT and PTT

In this section, the test statistics of the classical and robust Wald UT, RT, PT and PTT are defined.

4.1 The Classical and Robust Wald UT

In this section, the test statistics of the classical and robust Wald UT are defined.

(i) The Robust Wald UT (RWUT)

If β_2 is unspecified, the proposed test statistic for testing $H_0^* : \beta_1 = \beta_{01}$ against $H_A^* : \beta_1 \neq \beta_{01}$ is

$$RW_n^{UT} = \tilde{\gamma}^2 \left(\tilde{\beta}_1 - \beta_{01}\right)' Q_{n1}^* \left(\tilde{\beta}_1 - \beta_{01}\right) / \tilde{\sigma}^2, \tag{4.1}$$

where $Q_{n1}^* = Q_{n11} - Q_{n12}Q_{n22}^{-1}Q_{n21}$, in which $Q_{njk} = X'_{nj}X_{nk}$ for j, k = 1, 2. It follows from equation (3.6) that RW_n^{UT} follows a χ_r^2 (chi-squared distribution with r degrees of freedom) under H_0^* $as\ n \to \infty$.

(ii) The Classical Wald UT (CWUT)

The classical Wald UT is the nonrobust counterpart of the robust Wald UT and it is given as follows:

$$CW_n^{UT} = \left(\tilde{\beta}_1 - \beta_{01}\right)' Q_{n1}^* \left(\tilde{\beta}_1 - \beta_{01}\right) / \hat{\sigma}_0^2 \tag{4.2}$$

with $\tilde{\beta}_1$ is the first r rows of $\tilde{\beta}$. Note also that CW_n^{UT} follows as χ_r^2 under H_0^* as $n \to \infty$.

4.2 The Classical and Robust Wald RT

In this section, the test statistics of the classical and robust Wald RT are defined.

(i) The robust Wald RT (RWRT)

If $\beta_2 = \beta_{02}$ (specified), we find $Y_n = X_{n1}\beta_1 + X_{n2}\beta_{02} + e_n$, and the proposed test statistics for testing $H_0^* : \beta_1 = \beta_{01}$ against $H_A^* : \beta_1 \neq \beta_{01}$ is

$$RW_n^{RT} = \hat{\gamma}^2 \left(\hat{\beta}_1 - \beta_{01}\right)' Q_{n11} \left(\hat{\beta}_1 - \beta_{01}\right) / \tilde{\sigma}_2^2, \tag{4.3}$$

where $\tilde{\sigma}_2^2 = n^{-1} \times \sum \psi^2 \left(\frac{Y_i - X'_{1i}\hat{\beta}_1 - X'_2\beta_{02}}{S_n^{(2)}} \right)$ with $S_n^{(2)}$ is the scaled MAD of $\left(Y_i - X'_{1i}\hat{\beta}_1 - X'_{2i}\beta_{02}\right)$ and $\hat{\gamma} = \frac{1}{{}_nS_n^2} \times \sum \psi' \left(\frac{Y_i - X'_{1i}\hat{\beta}_1 - X'_{2i}\beta_{02}}{S_n^{(2)}} \right)$. It follows from equation (3.7) that $RW_n^{RT} \xrightarrow{d} \chi_r^2$ under $H_0^* : \beta_1 = \beta_{01}$ when $\beta_2 = \beta_{01}$ for large n.

(ii) The Classical Wald RT (CWRT)
The classical Wald RT is the nonrobust counterpart of the robust Wald RT and it is given as follows:

$$CW_n^{RT} = \left(\breve{\beta}_1^{\dagger} - \beta_{01}\right)' Q_{n11} \left(\breve{\beta}_1^{\dagger} - \beta_{01}\right) / \hat{\sigma}_2^2 \tag{4.4}$$

with $\breve{\beta}_1^{\dagger} = \left(X'_{n1}X_{n1}\right)^{-1} X'_{n1}Y_n^{**}$, where $Y_n^{**} = Y_n - X'_{n2}\beta_{02}$ and $\hat{\sigma}_2^2 = \sum \left(Y_i - X'_{1i}\breve{\beta}_1^{\dagger} - X'_{i2}\beta_{02}\right)^2 / n$. The CW_n^{RT} follows a χ_r^2 under H_0^* as $n \to \infty$.

4.3 The Classical and Robust Wald PT

The test statistics of the classical and robust Wald PT are defined below.

(i) The robust Wald PT (RWPT)
For the preliminary test on the β_2, the proposed test statistic for testing $H_0^{(1)} : \beta_2 = \beta_{02}$ against $H_A^{(1)} : \beta_2 \neq \beta_{02}$ is given by

$$RW_n^{PT} = \tilde{\gamma}^2 \left(\tilde{\beta}_2 - \beta_{02}\right)' Q_{n2}^* \left(\tilde{\beta}_2 - \beta_{02}\right) / \tilde{\sigma}^2 \tag{4.5}$$

where $Q_{n2}^* = Q_{n22} - Q_{n21}Q_{n11}^{-1}Q_{n12}$. It follows from equation (3.7) that $RW_n^{PT} \xrightarrow{d} \chi_t^2$ under $H_0^{(1)}$.

(ii) The Classical Wald PT (CWPT)

The classical Wald PT is the nonrobust counterpart of the robust Wald PT and it is given follow:

$$CW_n^{UT} = \left(\breve{\beta}_2 - \beta_{02}\right)' Q_{n2}^{*}\left(\breve{\beta}_2 - \beta_{02}\right)/\hat{\sigma}_0^2, \tag{4.6}$$

with $\breve{\beta}_2$ is the last t rows of $\breve{\beta}$. Then, $CW_n^{PT} \xrightarrow{d} \chi_t^2$ under $H_0^{(1)}$.

4.4 The Classical and Robust Wald PTT

If the null hypothesis of this pre-test is rejected, the UT is appropriate to test H_0^*, otherwise the RT is used to test H_0^*. The final test for testing $H_0^{(1)}$, following a pre-test on $H^{(1)}$, is termed as the pre-test test (PTT). The test statistic for testing H_0^* following a pre-test on $\boldsymbol{\beta}_2$ is a choice between RT and UT. The UT is used if $H_0^{(1)}: \beta = \beta_0$ is rejected and the RT is used if $H^{(1)}$ is accepted.

The asymptotic power functions of the classical and robust Wald UT, RT and PT are based on the univariate χ^2 probability distribution, but the asymptotic power function for the final PTT involves two bivariate χ^2 distributions (see Appendix B and C for details). Here the PT and RT are independent and PT and UT are correlated. So, the correlated bivariate non central χ^2 distribution is used to find the asymptotic power function of the PTT. Thus, the computation of the power of the PTT using the asymptotic power function involves the bivariate noncentral χ^2 probability integral. Instead of using directly the asymptotic power function formula to compute the power of the PTT, a Monte Carlosimulation method was used in this study.

5. Power Comparison using a Monte Carlo Simulation

To compare the performance of the tests, the analytical comparison is unrealistic. Instead a Monte Carlo simulation method is used to compare the power of the tests. In this section, the power of the classical and robust Wald UT, RT and PTT are obtained using a computer generated Monte Carlo experiment. The objectives of this section are to determine which of the UT, RT and PTT is better, and to compare the proposed robust Wald UT, RT and PTT to the classical Wald UT, RT and PTT, each under normality and a slight change to normality. A multiple liner regression model with three parameter $y_i = \theta_1 + \theta_2 x_{1i} + \beta x_{2i} + \varepsilon_i$ for i = 1, 2, ...,n was considered in the simulation. Here, take n = 100. The error terms ε_i, i = 1, 2, ...,n are generated randomly from (i) normal with mean 0 and variance 1, N (0, 1) (ii) 10% wild: First ε_i is generated from normal distribution with mean 0 and variance 1, then choose randomly 5% of the generated ε_i and multiply them by a scalar 10, and another 5% choose randomly 5% is multiply by a scalar -10. The observed values of the regression x_{1i} where generated randomly from a uniform distribution with minimum and maximum values of 0 and 1, and those of x_{2i} were from normal distribution with mean 1 and variance 1. We let $\theta_1 = \theta_{10} + n^{-\frac{1}{2}}\delta_1, \theta_2 = \theta_{20} + n^{-\frac{1}{2}}\delta_1; \quad \beta = \beta_0 + n^{-\frac{1}{2}}\delta_2, \quad with\ \delta_1, \delta_2 \geq 0, \theta_{10}, \theta_{20} \in \Re$ and then generate a random sample for selected values of $\theta_1, \theta_2\ and\ \beta$.

For selected values of δ_1 and δ_2, 5000 simulations were run in which a sample was drawn from normal or 10% wild distributions. Each of the tests was run with in each simulation and H_0^*:θ_1=θ_{10}, θ_2=θ_{20} was either rejected or not rejected at the 5% significance level. Size of the UT, RT and PTT is the probability of rejecting the null hypothesis H_0^*:θ_1= θ_{10}, θ_2= θ_{20} when it is true. First, we generate data set for δ_1=0. Then, each test statistic was computed for the dataset. We then find the proportion of tests rejecting the null hypothesis from 5000 simulated data sets, and then use it to estimate the size of the test for the UT and the RT. On the other hand, the power of the test is the probability of rejecting false H_0^*. We generate a different data set for an arbitrary positive value of δ_1 (eg.2, 4). As δ_1 moves away from 0, it is suspected that the power of test increases. To estimate the probability of rejecting the null hypothesis H_0^*, we find the proportion of rejecting the null hypothesis from 5000 simulated data sets when the true values of θ_1 and θ_2 are not respectively θ_{10} and θ_{20}, that is, when $\delta_1 > 0$.

For the PTT, the UT is used if $H_0^{(1)}$:β=β_0 is rejected and the RT is used if $H_0^{(1)}$ is accepted. So, the PTT is either the UT or RT depending on the outcome of the PT. The proportion of rejecting H_0^* from the UT or the RT following the result from the PT among 5000 generated data sets is taken as the probability of rejecting H_0^* for the PTT.

Since the classical and robust Wald UT, RT and PTT are defined based on the knowledge of β, it is of interest to compare the size and power of the tests by plotting them against δ_2, where $\delta_2 = \sqrt{n}\left(\beta - \beta_0\right)$. Figure1 (in Appendix D) shows the size and power of the classical and robust Wald UT, RT and PTT, when the distribution of the error term in the multiple linear regression model is $N(0,1)$ and 10% wild, and sample size is n =100 for selected values of δ_1. Our first aim is to determine which of the UT, RT and PTT is better, both in the size and power of the test. Although the classical and robust Wald RT have the largest power in comparison to those of the UT and PTT, it also has the largest size as δ_2 grows larger. The RT is defined when β=β_0, and it is as expected that the size of RT increases as $\delta_2 = \sqrt{n}\left(\beta - \beta_0\right)$ increases. On the other hand, the size and power of the UT are constant regardless the value of δ_2. This is because UT treats β as a nuisance parameter. In comparison to RT, the UT has the smallest size, but also the smallest power for small δ_2 because β is not specified at the nullhypothesis in the specification of the UT. The PTT is either the UT or RT depending on the PT. Thus it is a compromise between the two tests. The PTT behaves similar to the RT for small δ_2, that is, when PT accepts $H_0^{(1)}$. On the other hand, it behaves similar to the UT when $H_0^{(1)}$ is rejected using the PT. Thus, for a larger value of δ_2, it behaves similar to the UT. The PTT is better than the RT in terms of size and it is better than the UT in terms of power when δ_2 is small. Although the prior information on the β vector is uncertain, there is a high possibility that its true value is quite close to the suspected value. Therefore, the study on the behaviour of the three tests when δ_2 is small is more realistic.

It is of interest to see the effect of wild observations in the data on the classical and robust Wald tests performance. Figure1(in Appendix D) depicts that the power of classical Wald UT, RT and PTT are about the same as those of the robust Wald UT, RT and PTT when the distribution of the error term is normal (see Figures 1(a), (c) and (e)). However, under a slight change to normality, the classical test lost its power and the robust Wald PTT has shown a remarkable performance in terms size and power compared to the classical Wald tests when the error term is 10% wild (see Figures 1(b), (d) and (f)). The robust Wald PTT has shown some robustness property under a slight change to normality assumption through this simulation example.

The performance of a test depends on the sample size. Figure 2 (in Appendix D) shows the power of the classical and robust Wald UT, RT and PTT for data generated from normal or
10% wild distributions when $\delta_1 = 2$ and for some selected sample sizes n = 40, 60 and 120. The proportion of rejecting H_0^* is plotted against Δ_β, where $\Delta_\beta = \beta - \beta_0 = \delta_2 / \sqrt{n}$. As n grows larger, the performance of the tests differ. When the distribution of the error term is normal, the power of both classical and robust Wald UTs are same and constant but increase as n grows larger. The power of the classical and robust Wald RT increase as Δ_β increases and are larger for data with a larger sample size than that with a smaller sample size. The power of the PTT is better than that of the UT and is larger for data with a larger sample size when Δ_β is small, i.e. when the suspected value is close to the true β. When the distribution of the error term is10% wild, the classical and robust Wald tests performance differ more as n gets smaller. Again, the powers of the tests are larger for data with larger sample sizes. The power of all the classical UT, RT and PTT tests are smaller than those of the robust Wald tests for smaller Δ_β, i.e. when the suspected value is close to the true β. Even for a larger value of Δ_β, i.e. in case when the suspected value is quite far away from the true β, the robust Wald tests have more power than the classical tests for data with a smaller n that those with a larger sample size. As sample size grows larger, the robust Wald tests are more insensitive to

wild observations (outliers) and have closer power performance to those tests in the normal case. The classical Wald tests fail to maintain similar power as they have in the normal case when the sample size is small and in the presence of wild observations.

In the next section, the test statistics of the UT, RT and PTT are computed for the Olympic athletes data set.

6. Application on Data

In this section, the proposed robust Wald UT, RT and PT were used on a set of real life data to view the effect of pre-testing (PT) on the final test (PTT) and the effectiveness of the robust Wald test compared to its classical counterpart. For this illustration, we used 2012 Olympic athletes data which can be accessed from the Guardian website, *http://www.theguardian.com/sport/datablog/2012/aug/07/olympic-2012-athletes-age-weight-height*.

The data set contains several variables related to the athletes from 205 countries, including place of birth, height, weight, age, type of sport, etc. However, we use the data set on Australia athletes and focus on three variables weight, height and age. The height (in cm) and age (in year) are the independent variables and weight (in kilogram) is the response variable. The regression model is as follows:

$$\text{weight}_i = \beta_0+\beta_1\,\text{height}_i + \beta_2\,\text{age}_i + e_i. \tag{6.7}$$

The quantile-quantile normal plot and scatter plot of the standardized residuals (see Figure 3, as in Appendix D) from the least-squares fitted model revealed several outliers in the data. The least squares estimates of β_0, β_1 and β_2 are 138.8, 0.587, and -0.1178, respectively. Table 1 gives the test results for two hypothesis: (i) $H_0^*: \beta_0 = 140;\ \beta_1 = 0.6$ (using RT and UT), and (ii) $H_0^{(1)}: \beta_2 = 0$ (using PT).

In the case of the robust Wald test, the age is not significant at the 5 % level for the PT (P=0.052). Thus, PTT becomes the RT, and we then reject at the 5% level (P<0.001). On the contrary, age becomes significantly important in the case of the classical Wald test at the 5% significance level (P = 0.044), so we use the UT as the final test, and we cannot reject H_0^* (P=0.323).

So the classical Wald PTT cannot reject H_0^* using the classical Wald UT. But under the robust Wald test, the PTT becomes the RT leadings to the rejection of H_0^*. Since the robust Wald PTT has more power than the classical Wald PTT under a slight departure from normality assumption, we decide to reject H_0^* at the 5% significance level.

7. Concluding remarks

In this paper, we have introduced robust Wald UT, RT and PT in the context of multiple regression model in the preliminary testing framework. Theorem 3.1 shows it is clear that there is a correlation between the two components of the test statistics, namely the robust Wald UT and PT, but no correlation between that of the robust Wald RT and PT from the structure of the covariance matrix in equations (3.6) and (3.7). Since robust Wald PTT is either UT or RT depending on the outcome of the PT, its asymptotic distribution is determined from the distribution of the test statistics for the UT, RT and PT. It is found that, the asymptotic distribution of the robust Wald PTT is a bivariate noncentral chi-squared distribution with the noncentrality parameters of the robust Wald UT, RT and PT. The asymptotic joint distribution of the test statistics under the alternative hypothesis is used to obtain the asymptotic power function (See Appendix B and C for details).

For the purpose of comparing the power of the competing tests, the Monte Carlo experiment is used. The asymptotic power of the PTT is obtained as the proportion of the rejection of the UT or the RT in N replicated samples, following the result from the PT. From the simulation, both the classical and robust Wald tests are preferred when normality assumption is satisfied, as both tests have similar performance in terms of size and power of the test. However, in the presence of contamination or wild observations in the data, the classical Wald test lose its power. On the other hand, the power of the robust Wald test is not affected by the wild observations in the data. Thus, the robust Wald test has shown a remarkable robustness property under a slight departure from normality assumption.

The robust Wald RT has the highest power compared to those of the UT and PTT, but it also has the largest size. On the contrary, the robust Wald UT's size is the smallest, but its power is also the smaller expect when $\lambda_1 = n^{\frac{1}{2}}(\beta_1 - \beta_{01})$ or $\lambda_2 = n^{\frac{1}{2}}(\beta_1 - \beta_{02})$ is large. So, both robust Wald UT and RT fail to attain the lowest size and the highest power criteria. The robust Wald PTT's size is smaller than that of the robust Wald RT. Its power is also higher than the robust Wald UT, with the exception of very large values of $\boldsymbol{\lambda}_1$ or $\boldsymbol{\lambda}_2$. Consequently, if the prior information on the value of $\boldsymbol{\beta}_2$ is not far from its true value, that is, $\boldsymbol{\lambda}_2$ is near $\mathbf{0}$ (small or moderate) difference the robust Wald PTT's size is smaller than that of the RT, and its power is higher than that of the UT. Thus, the robust Wald PTT is a better choice among the three tests regardless the normality assumption is violated or not. Since the prior information is given by experienced experts in the field or from previous studies, the value of $\boldsymbol{\lambda}_2$ should not be far from $\mathbf{0}$, even though it may not be $\mathbf{0}$, and therefore the robust Wald PTT is preferable over those of the UT and RT.

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A Appendix A

Proof of Theorem 3.1(i)

Following Theorem 5.5.1 of Jurečková and Sen (1996), we find that $\tilde{\boldsymbol{\beta}}$ admits the asymptotic representation

$$\begin{pmatrix} \tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1 \\ \tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2 \end{pmatrix} = (n\gamma)^{-1} \begin{pmatrix} \mathbf{Q}_{n11} & \mathbf{Q}_{n12} \\ \mathbf{Q}_{n21} & \mathbf{Q}_{n22} \end{pmatrix}^{-1} \sum_{i=1}^{n} \mathbf{X}_i \psi(u_i/s) + R_n \tag{1}$$

with $R_n = O_p(n^{-1})$ and

$$\begin{pmatrix} \mathbf{Q}_{n11} & \mathbf{Q}_{n12} \\ \mathbf{Q}_{n21} & \mathbf{Q}_{n22} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{Q}_{n1}^{\star\,-1} & -\mathbf{Q}_{n11}^{-1}\mathbf{Q}_{n12}\mathbf{Q}_{n2}^{\star\,-1} \\ -\mathbf{Q}_{n22}^{-1}\mathbf{Q}_{n21}\mathbf{Q}_{n1}^{\star\,-1} & \mathbf{Q}_{n2}^{\star\,-1} \end{pmatrix},$$

where $\mathbf{Q}_{n1}^{\star} = \mathbf{Q}_{n11} - \mathbf{Q}_{n12}\mathbf{Q}_{n22}^{-1}\mathbf{Q}_{n21}$ and $\mathbf{Q}_{n2}^{\star} = \mathbf{Q}_{n22} - \mathbf{Q}_{n21}\mathbf{Q}_{n11}^{-1}\mathbf{Q}_{n12}$.

Since $(\mathbf{Q}_{11}^{-1}\mathbf{Q}_{12}\mathbf{Q}_{2}^{\star-1})' = \mathbf{Q}_{22}^{-1}\mathbf{Q}_{21}\mathbf{Q}_{1}^{\star-1}$ and $\mathrm{Var}\,(\mathbf{X}_i\psi(u_i/s)) = n\sigma^2\mathbf{Q}_n$, the proof of part (i) follows after some algebra.

Proof of Theorem 3.1(ii)

It follows from (1) that

$$\tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1 = \frac{1}{n\gamma}\left\{\mathbf{Q}_{n1}^{\star\,-1}\sum \mathbf{X}_{1i}\psi\left(\frac{u_i}{s}\right) - \mathbf{Q}_{n11}^{-1}\mathbf{Q}_{n12}\mathbf{Q}_{n2}^{\star\,-1}\sum \mathbf{X}_{2i}\psi\left(\frac{u_i}{s}\right)\right\} + R_n \tag{2}$$

and

$$\tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2 = \frac{1}{n\gamma}\left\{-\mathbf{Q}_{n22}^{-1}\mathbf{Q}_{n21}\mathbf{Q}_{n1}^{\star\,-1}\sum \mathbf{X}_{1i}\psi\left(\frac{u_i}{s}\right) + \mathbf{Q}_{n2}^{\star\,-1}\sum \mathbf{X}_{2i}\psi\left(\frac{u_i}{s}\right)\right\} + R_n. \tag{3}$$

On the other hand, we admits that

$$\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1 = \frac{1}{n\gamma}\left\{\mathbf{Q}_{n11}^{-1}\sum \mathbf{X}_{1i}\psi(u_i/S)\right\} + R_n \tag{4}$$

when $\boldsymbol{\beta}_2 = \mathbf{0}$. Then, (2), (3) and (4) imply that

$$\mathbf{Q}_{n22}^{-1}\mathbf{Q}_{n21}\mathbf{Q}_{n1}^{\star\,-1}\mathbf{Q}_{n11}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1) = -(\tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2) + \frac{1}{n\gamma}\mathbf{Q}_{n2}^{\star\,-1}\sum \mathbf{X}_{2i}\psi\left(\frac{u_i}{s}\right) + R_n \tag{5}$$

and

$$\mathbf{Q}_{n1}^{\star\,-1}\mathbf{Q}_{n11}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1) = (\tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1) + \frac{1}{n\gamma}\mathbf{Q}_{n11}^{-1}\mathbf{Q}_{n12}\mathbf{Q}_{n2}^{\star\,-1}\sum \mathbf{X}_{2i}\psi\left(\frac{u_i}{s}\right) + R_n \tag{6}$$

after some algebra. Then solving equations (5) and (6) take us to

$$(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1) = (\tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1) + \mathbf{Q}_{n11}^{-1}\mathbf{Q}_{n12}(\tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2) + R_n. \tag{7}$$

It follows from equation (7) that

$$\begin{pmatrix} \hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1 \\ \tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{I}_r & \mathbf{Q}_{n11}^{-1}\mathbf{Q}_{n12} \\ 0 & \mathbf{I}_s \end{pmatrix} \begin{pmatrix} \tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1 \\ \tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2 \end{pmatrix} + R_n. \tag{8}$$

Thus the asymptotic distribution of $\sqrt{n}\begin{pmatrix} \hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_1 \\ \tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_2 \end{pmatrix}$ under H_0 is bivariate normal with zero mean vector and covariance matrix

$$\frac{\sigma^2}{\gamma^2}\begin{pmatrix} \mathbf{I}_r & \mathbf{Q}_{11}^{-1}\mathbf{Q}_{12} \\ 0 & \mathbf{I}_s \end{pmatrix}\begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{pmatrix}^{-1}\begin{pmatrix} \mathbf{I}_r & \mathbf{Q}_{11}^{-1}\mathbf{Q}_{12} \\ 0 & \mathbf{I}_s \end{pmatrix}' = \frac{\sigma^2}{\gamma^2}\begin{pmatrix} \mathbf{Q}_{11}^{\;-1} & 0 \\ 0 & \mathbf{Q}_{2}^{\star-1} \end{pmatrix}. \tag{9}$$

B Appendix B

In this section, two theorems related to the distributions of the M-estimators and test statistics are derived under a sequence of local alternatives.

Theorem B.1 *Let $\{K_n\}$ be a sequence of local alternatives, where*

$$K_n : (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) = (\boldsymbol{\beta}_{01} + n^{-\frac{1}{2}}\boldsymbol{\lambda}_1, \boldsymbol{\beta}_{02} + n^{-\frac{1}{2}}\boldsymbol{\lambda}_2), \tag{10}$$

with $\boldsymbol{\lambda}_1 = n^{\frac{1}{2}}(\boldsymbol{\beta}_1 - \boldsymbol{\beta}_{01}) > \mathbf{0}$ and $\boldsymbol{\lambda}_2 = n^{\frac{1}{2}}(\boldsymbol{\beta}_2 - \boldsymbol{\beta}_{02}) > \mathbf{0}$ are (fixed) real numbers. Under $\{K_n\}$, asymptotically,

$$\sqrt{n}\begin{pmatrix} \tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01} \\ \tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_{02} \end{pmatrix} \xrightarrow{d} N_p\left[\begin{pmatrix} \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \end{pmatrix}, \frac{\sigma^2}{\gamma^2}\begin{pmatrix} \boldsymbol{Q}_1^{\star -1} & \boldsymbol{Q}_{12}^{\star} \\ \boldsymbol{Q}_{21}^{\star} & \boldsymbol{Q}_2^{\star -1} \end{pmatrix}\right], \tag{11}$$

$$\sqrt{n}\begin{pmatrix} \hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01} \\ \tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_{02} \end{pmatrix} \xrightarrow{d} N_p\left[\begin{pmatrix} \boldsymbol{\lambda}_1 + \boldsymbol{Q}_{11}^{-1}\boldsymbol{Q}_{12}\boldsymbol{\lambda}_2 \\ \boldsymbol{\lambda}_2 \end{pmatrix}, \frac{\sigma^2}{\gamma^2}\begin{pmatrix} \boldsymbol{Q}_{11}{}^{-1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{Q}_2^{\star -1} \end{pmatrix}\right]. \tag{12}$$

Proof The proof of this Theorem is obtained directly from equations (3.6) and (3.7) using the contiguity probability measures (Hájek et al. 1999). ■

Theorem B.2 *Under $\{K_n\}$, asymptotically (RW_n^{RT}, RW_n^{PT}) are independently distributed as bivariate noncentral chi-squared distribution with (r, t) degrees of freedom and (RW_n^{UT}, RW_n^{PT}) are distributed as correlated bivariate noncentral chi-squared distribution with (r, t) degrees of freedom and noncentrality parameters,*

$$\begin{aligned}
\theta^{UT} &= \gamma^2(\boldsymbol{\lambda}_1'\boldsymbol{Q}_1^{\star}\boldsymbol{\lambda}_1)/\sigma^2, && (13)\\
\theta^{RT} &= \gamma^2(\boldsymbol{\lambda}_1'\boldsymbol{Q}_{11}\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_1'\boldsymbol{Q}_{12}\boldsymbol{\lambda}_2 + \boldsymbol{\lambda}_2'\boldsymbol{Q}_{21}\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2'\boldsymbol{Q}_{21}\boldsymbol{Q}_{11}^{-1}\boldsymbol{Q}_{12}\boldsymbol{\lambda}_2)/\sigma^2, && (14)\\
\theta^{PT} &= \gamma^2(\boldsymbol{\lambda}_2'\boldsymbol{Q}_2^{\star}\boldsymbol{\lambda}_2)/\sigma^2. && (15)
\end{aligned}$$

Proof From Theorem B.1, we find that

$$n^{-\frac{1}{2}}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01}) \xrightarrow{d} N_r\left(\boldsymbol{\lambda}_1 + \boldsymbol{Q}_{11}^{-1}\boldsymbol{Q}_{12}\boldsymbol{\lambda}_2, (\sigma^2/\gamma^2)\boldsymbol{Q}_{11}^{-1}\right) \tag{16}$$

and using Theorem 1.4.1 of Muirhead (1982)

$$RW_n^{RT} = \hat{\gamma}^2(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01})'\mathbf{Q}_{n11}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01})/\hat{\sigma}^2 \tag{17}$$

is a chi-squared distribution with r degrees of freedom and noncentrality parameter

$$\theta^{RT} = \gamma^2(\boldsymbol{\lambda}_1 + \boldsymbol{Q}_{11}^{-1}\boldsymbol{Q}_{12}\boldsymbol{\lambda}_2)'\boldsymbol{Q}_{11}(\boldsymbol{\lambda}_1 + \boldsymbol{Q}_{11}^{-1}\boldsymbol{Q}_{12}\boldsymbol{\lambda}_2)/\sigma^2 \tag{18}$$

which is simplified as $\theta^{RT} = \frac{\gamma^2}{\sigma^2}(\boldsymbol{\lambda}_1'\boldsymbol{Q}_{11}\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_1'\boldsymbol{Q}_{12}\boldsymbol{\lambda}_2 + \boldsymbol{\lambda}_2'\boldsymbol{Q}_{21}\boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2'\boldsymbol{Q}_{21}\boldsymbol{Q}_{11}^{-1}\boldsymbol{Q}_{12}\boldsymbol{\lambda}_2)$ after some algebra. In the same manner, the other two noncentrality parameters θ^{UT} and θ^{PT} are obtained.

There is no correlation between RW_n^{RT} and RW_n^{PT} because the covariance matrix between $\sqrt{n}(\hat{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01})$ and $\sqrt{n}(\tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_{02})$ is a zero matrix.

Note that for any two variables Z_1 and Z_2 that follow a bivariate normal with mean 0 and covariance matrix $\begin{pmatrix} \sigma_{z1}^2 & \rho_z\sigma_{z1}\sigma_{z2} \\ \rho_z\sigma_{z1}\sigma_{z2} & \sigma_{z2}^2 \end{pmatrix}$, the correlation coefficient between $U = Z_1^2/\sigma_{z1}^2$ and $V = Z_2^2/\sigma_{z2}^2$ is ρ_z^2 (cf. Joarder 2006).

From Theorem 3.1, the covariance matrix of two vectors $\sqrt{n}(\tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01})$ and $\sqrt{n}(\tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_{02})$ is a nonzero matrix of size r by s. The (i, j)th element of this covariance matrix is the covariance of the ith element of vector $\sqrt{n}(\tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01})$ and the jth element of vector $\sqrt{n}(\tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_{02})$. Denote $\rho_k^\diamond$ as the correlation coefficient for any two different elements of the augmented vector $(\sqrt{n}(\tilde{\boldsymbol{\beta}}_1 - \boldsymbol{\beta}_{01}), \sqrt{n}(\tilde{\boldsymbol{\beta}}_2 - \boldsymbol{\beta}_{02}))$, following Joarder (2006), the correlation between RW_n^{UT} and RW_n^{PT} is $\sum_{k=1}^{p} \rho_k^{\diamond 2}/p$. ■

The asymptotic distributions of test statistics given in this section are used to derive the asymptotic power functions of the test statistics in the following section.

C Appendix C

Using results in Theorem B.2, under $\{K_n\}$, the asymptotic power function for the UT, RT and PT is

$$\Pi^{UT}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \lim_{n\to\infty} \Pi_n^{UT}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \lim_{n\to\infty} P(RW_n^{UT} > \ell_{n,\alpha_1}^{UT}|K_n) = 1 - G_r(\chi^2_{r,\alpha_1}; \theta^{UT}), \quad (19)$$

$$\Pi^{RT}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \lim_{n\to\infty} \Pi_n^{RT}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \lim_{n\to\infty} P(RW_n^{RT} > \ell_{n,\alpha_2}^{RT}|K_n) = 1 - G_r(\chi^2_{r,\alpha_2}; \theta^{RT}), \quad (20)$$

$$\Pi^{PT}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \lim_{n\to\infty} \Pi_n^{PT}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \lim_{n\to\infty} P(RW_n^{PT} > \ell_{n,\alpha_3}^{PT}|K_n) = 1 - G_s(\chi^2_{s,\alpha_3}; \theta^{PT}), \quad (21)$$

respectively, with $G_k(\chi^2_{k,\alpha}; \theta)$ is the cumulative distribution function of the noncentral chi-squared distribution with k degrees of freedom and noncentrality parameter θ, and level of significance α. Here, $\chi^2_{k,\alpha}$ is the upper $100\alpha\%$ critical value of a central chi-squared distribution and $\ell_{n,\alpha_1}^{UT} \to \chi^2_{r,\alpha_1}$ under $H_0^\star$, $\ell_{n,\alpha_2}^{RT} \to \chi^2_{r,\alpha_2}$ under $H_0^\star$ when $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_{02}$, and $\ell_{n,\alpha_3}^{PT} \to \chi^2_{s,\alpha_3}$ under $H_0^{(1)}$. Since the PTT is a choice between RT and UT, define the power function of the PTT as

$$\begin{aligned} & \Pi_n^{PTT}(\boldsymbol{\beta}_1) \\ = \; & E(I\left[(RW_n^{PT} < \ell_{n,\alpha_3}^{PT}, RW_n^{RT} > \ell_{n,\alpha_2}^{RT}) \text{ or } (RW_n^{PT} > \ell_{n,\alpha_3}^{PT}, RW_n^{UT} > \ell_{n,\alpha_1}^{UT})\right] |\boldsymbol{\beta}_1) \end{aligned} \quad (22)$$

where ℓ_{n,α_3}^{PT} is the critical value of RW_n^{PT} at the α_3 level of significance and $I(A)$ is an indicator function of the set A which takes value 1 if A occurs, otherwise it is 0. The size of the PTT is obtained by substituting $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_{01}$ in equation (22). For testing $H_0^\star$ following a pre-test on $\boldsymbol{\beta}_2$, using equation (22) and the results of Theorem B.2, the asymptotic power function for the PTT under $\{K_n\}$ is given by

$$\begin{aligned} & \Pi^{PTT}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) \\ = \; & \lim_{n\to\infty} P(RW_n^{PT} \le \ell_{n,\alpha_3}^{PT}, RW_n^{RT} > \ell_{n,\alpha_2}^{RT}|K_n) + \lim_{n\to\infty} P(RW_n^{PT} > \ell_{n,\alpha_3}^{PT}, RW_n^{UT} > \ell_{n,\alpha_1}^{UT}|K_n) \\ = \; & G_s(\chi^2_{s,\alpha_3}; \theta^{PT})\{1 - G_r(\chi^2_{r,\alpha_2}; \theta^{RT})\} + \int_{\chi^2_{r,\alpha_1}}^{\infty} \int_{\chi^2_{s,\alpha_3}}^{\infty} \phi^\star(w_1, w_2) dw_1 dw_2, \end{aligned} \quad (23)$$

where, $\phi^\star(\cdot)$ is the density function of a bivariate noncentral chi-squared distribution with probability integral given by

$$
\begin{aligned}
&\int_{\chi^2_{r,\alpha_1}}^{\infty}\int_{\chi^2_{t,\alpha_3}}^{\infty} \phi^\star(w_1,w_2)dw_1dw_2 \\
=\ & \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{\delta_1=0}^{\infty}\sum_{\delta_2=0}^{\infty}(1-\rho^{\star 2})^{(r+t)/2}\frac{\Gamma(\frac{r}{2}+j)}{\Gamma(\frac{r}{2})j!}\frac{\Gamma(\frac{t}{2}+k)}{\Gamma(\frac{t}{2})k!}\rho^{\star 2(j+k)} \\
&\times\left[1-\gamma^\star\left(\frac{r}{2}+j+\delta_1,\frac{\chi^2_{r,\alpha_1}}{2(1-\rho^{\star 2})}\right)\right]\left[1-\gamma^\star\left(\frac{t}{2}+k+\delta_2,\frac{\chi^2_{t,\alpha_3}}{2(1-\rho^{\star 2})}\right)\right] \\
&\times\frac{e^{-\theta^{UT}/2}(\theta^{UT}/2)^{\delta_1}}{\delta_1!}\frac{e^{-\theta^{PT}/2}(\theta^{PT}/2)^{\delta_2}}{\delta_2!},
\end{aligned}
\tag{24}
$$

with (r,t) degrees of freedom, noncentrality parameters, θ^{UT} and θ^{PT}, correlation coefficient, $\rho^{\star 2}$, with $-1<\rho^\star<1$ and the incomplete gamma function, $\gamma^\star(v,d)=\int_0^d x^{v-1}e^{-x}/\Gamma(v)dx$. See Yunus and Khan (2011c) for details of the bivariate probability integral. Let $\rho^{\star 2}=\sum_{k=1}^{p}\frac{1}{p}\rho_k^{\diamond 2}$, the mean correlation, where $\rho_k^\diamond$ is the correlation coefficient for any two different elements of the augmented vector $[\ n^{-\frac{1}{2}}(\hat{\boldsymbol{\beta}}_1-\boldsymbol{\beta}_{01}),\ \ n^{-\frac{1}{2}}(\tilde{\boldsymbol{\beta}}_2-\boldsymbol{\beta}_{02})\ ]$ in equation (3.6).

Table 1: Test results from weight-height-age relationship of Australian Olympic athletes

	robust Wald test			classical Wald test		
Hypothesis	Test	χ^2	p-value	Test	χ^2	p-value
$H_0^\star:\beta_0=140,$	UT	5.35	0.069	UT	2.27	0.323
$\beta_1=0.6,$	RT	216.91	<0.001	RT	223.73	<0.001
$H_0^{(1)}:\beta_2=0$	PT	3.78	0.052	PT	4.07	0.044

D Appendix D - Figures

1. Figure 1. Size and power of the classical and robust Wald UT, RT and PTT for simulated data sized $n=100$ with normal or 10% wild errors and for selected values of δ_1.
2. Figure 2. Power of the classical and robust Wald UT, RT and PTT for simulated data sized $n=40,60,100$ with normal or 10% wild errors when $\delta_1=2$.
3. Figure 3. Diagnostics from weight-height-age linear relationship for n =399 Australian Olympic athletes.

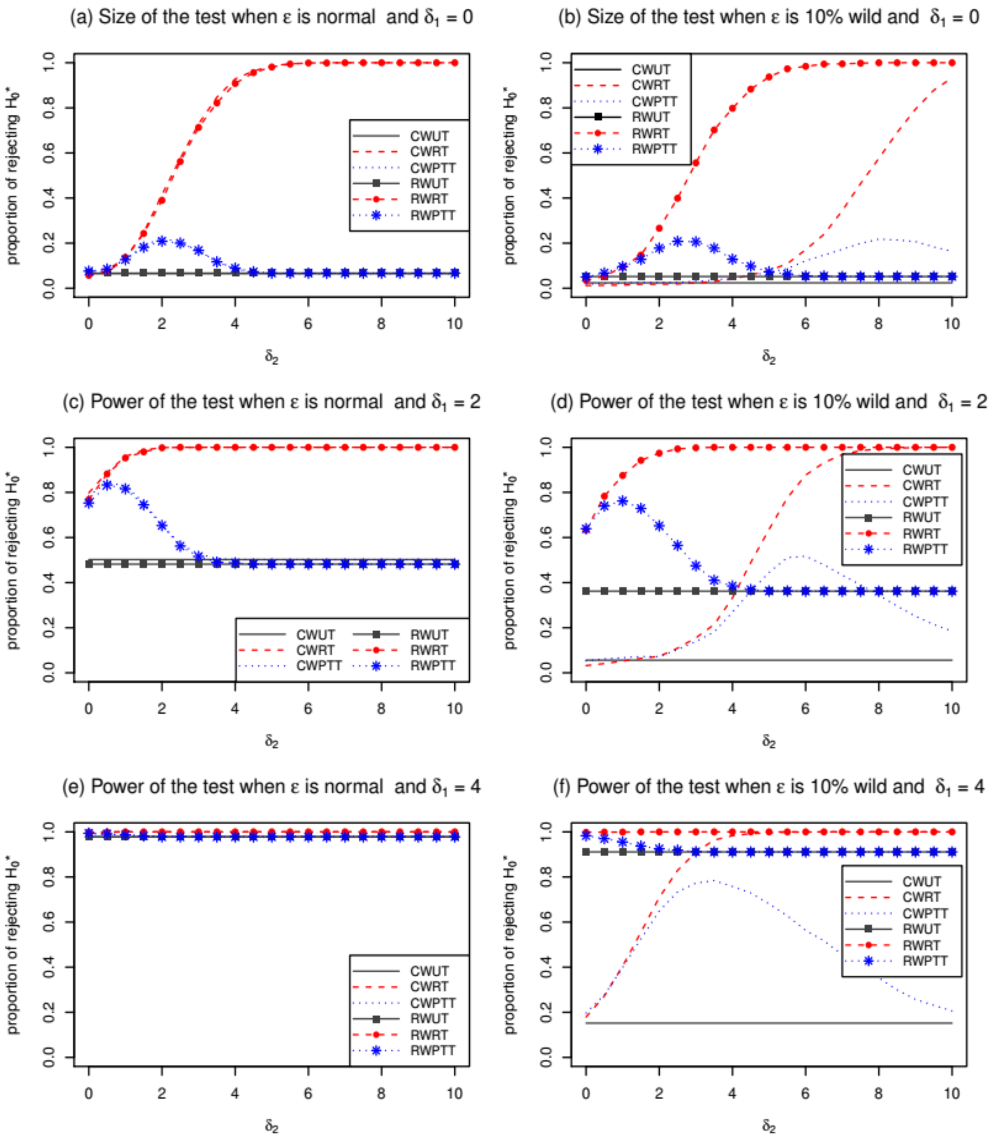

Figure 1: Size and power of the classical and robust Wald UT, RT and PTT for simulated data sized $n = 100$ with normal or 10% wild errors and for selected values of δ_1.

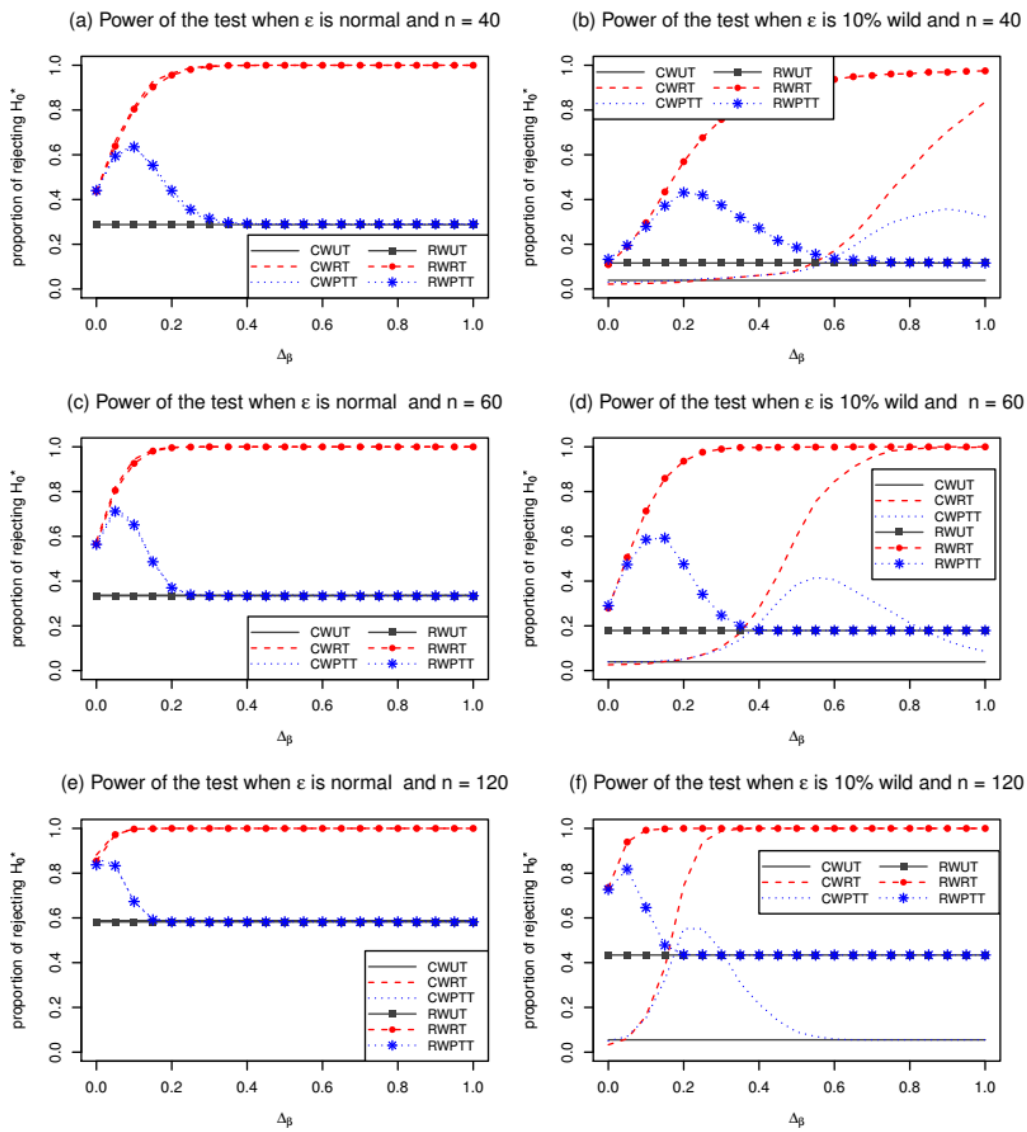

Figure 2: Power of the classical and robust Wald UT, RT and PTT for simulated data sized $n = 40, 60, 100$ with normal or 10% wild errors when $\delta_1 = 2$.

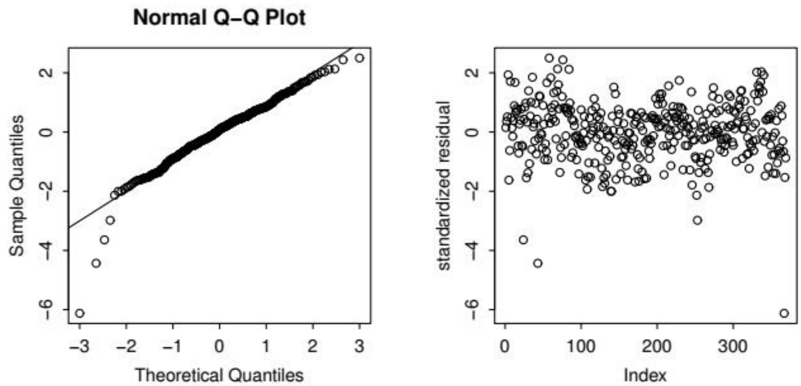

Figure 3: Diagnostics from weight-height-age linear relationship for n =399 Australian Olympic athletes.

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The Effectiveness of Health Literacy Intervention (Educational Booklet) on Chronic Neck Pain Management: Experiment Based on Repeated Measurements

M. Mazibar Rahman*
Department of Statistics Jahangirnagar University Savar, Dhaka-1342, Bangladesh

Mustafa Amimul Ehsan Siddique
Department of Statistics, Jahangirnagar University, Savar, Dhaka, Bangladesh

Abstract

Health literacy represents a constellation of skills necessary for people to function effectively in the health care environment and act appropriately on health care information. The study explores the effectiveness of educational booklet with conventional physiotherapy compare to only conventional physiotherapy for chronic neck pain. 8 chronic neck pain patients were randomly assigned to conventional physiotherapy group and 8 patients to conventional physiotherapy group. Unrelated "t" test was used to compare the results of ROM and Pain. Repeated measures analyses of variance (ANOVA) were used. Mean age of the participants of experimental and control group were consequence 41 &37 years. In Experimental group, Mean difference in different occasions found more reduction in pain than control group. which were 0.20, 0.50, 0.24 respectively in pretest mid test and posttest. There were significant effects of booklet on pain in the experimental group. There was also significant effect on time on pain reduction. This experimental study shows that there was significant difference between educational booklet with conventional physiotherapy and only conventional physiotherapy for patients with chronic neck pain.

Key words: Health Literacy Intervention, Educational booklet, Chronic Neck Pain management

1. Introduction

Musculoskeletal problems are one of the most common health problems in Bangladesh as well as in other countries. Taking a broad spectrum there are thousands of different musculoskeletal problems, including neck pain. Neck pain and other related disorders are very common all over the world. It is the eighth leading cause of disability in the United States and fourth worldwide (Sberman et al., 2014). Neck pain is the pain which may be experienced anywhere from the base ofthe skull at ear level to the upper part of the back or shoulder (Sabeen et al., 2013). Itis estimated that in every year 30-50% of adults experience a significant form of neck pain (Mantyselka et al., 2010). On general health showed that 15% of patient reported about grade 2 to 4 neck pain (Manchikanti et al., 2013). It is also a common symptom among the middle aged population and it has been shown that 24% of males and 37% of females suffer from neck pain (Mantyselka et al., 2010). It has also been shown that neck pain is most common between the ages of 40-50 with a reported

* Corresponding author: mazibm@juniv.edu

prevalence of 83 people per 100,000 populations (Physiopedia, 2013). In Sweden, females aged between 35-44 years had a higher risk of having long and medium- term neck pain and ≥ 65 aged males had a higher risk of having long and medium term neck pain symptoms than males aged between35-44 (Linder et al., 2012). Work related neck disorders are common problems in office workers, especially among those who are intensive computer users. (Sabeen et al., 2013).

Consequently, neck pain has been a source of disability which may require substantial health care resources and treatments (Cheng & Huang, 2014). Physiotherapy has a wide ranging role at all stages of neck pain to help the patient return early to normal activities (Moffett & Mclean, 2006). Physiotherapy usually forms part of the treatment offered for patients with neck pain and it includes specific exercise programs like Mckenzie approach, manual therapy (spinal manipulation and mobilization), traditional massage, physical modalities and proper patient education (Moffett & Mclean, 2006).

A review of the literature shows that patient education in the form of a brief intervention can be effective for chronic back pain (Moffett & Mclean, 2006). So this study focuses on the effectiveness of an educational booklet with a brief intervention for chronic neck pain.

Health literacy interventions that use written and print-based materials designed to be easy to read and understand have been shown to increase health-related knowledge among children and adults more than traditional materials. (Darren 2009) Primary care-based group education programs designed to be sensitive to health literacy limitations also appear to increase knowledge among adults. (Brega et al. 2013)

This research is a quantitative evaluation of the educational booklet used along with conventional physiotherapy for chronic neck pain. To identify the efficacy and effectiveness of this treatment approach, two measuring tools were used. First the Numeric Pain Rating scale (NPR) was used as a measurement tool for measuring the pain intensity in several functioning positions, and second the Goniometer was used to measure the range of motion. The main objectives of this study are

- To find out the effectiveness of an educational booklet for chronic neck pain.
- To evaluate the intensity of pain at resting position after using the educational booklet.
- To measure the intensity of pain during activity after using the educational booklet.
- To calculate the intensity of pain at different functional position (Sitting, Standing, Walking, sleeping, neck turning, neck bending).

2. Methodology

An experimental hypothesis predicts a relationship between two variables. So the study adopts a true repeated measurement experiment between different treatment designs. Conventional physiotherapy used together with an educational booklet was applied to the experimental group, and conventional Physiotherapy alone was applied to the control group. After the manipulation of the independent variables, the outcomes were compared. A pretest (before intervention), mid-test and posttest (after intervention) were administered with each participant of both groups to compare the pain effects before, middle of treatment and after the treatment. Study area was Physiotherapy unit of, CRP Manikganj.

Study sampling 16 patients with neck pain were selected through random sampling. A total of 16 patients were included in this study, among them 8 patients were selected for the experimental group and the rest were selected for the control group. Patients were selected through some exclusion and inclusion criteria.

Inclusion criteria

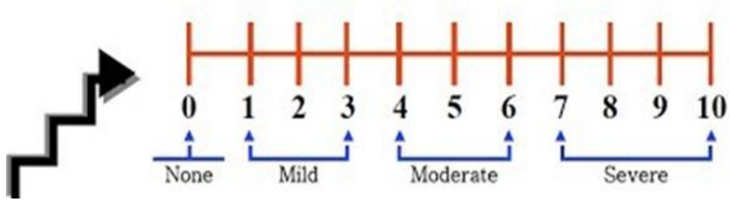
- Age between 25 to 50 years.
- Patients who have chronic neck pain.
- Both male and female are included.
- Patients who are literate.
- Patients who have postural and derangement problems
- Those who showed willingness to participants.

Exclusion criteria

- Patients who have pathological problems like tumors
- Neck deformity
- Neck dysfunction
- Severe trauma
- Adhesive capsulitis or other shoulder problems
- Post-operative conditions
- Patients who are not willing.

3. Methods of data collection

Numerical Pain Rating Scale (NPRS)-The NPRS was used for measuring the pain intensity in several function positions. Patients were asked to indicate the intensity of current, best and worst levels of pain using an 11-point scale, ranging from 0 (no pain) to 10 (worst pain imaginable) (Cleland et al.,2008).



Goniometer (Double-Armed) –A goniometer was used for assessing a joint Range of Motion (ROM).

Repeated Measures ANOVA

Pain Reduction in Control Group in Different Occasions

Subjects	Total pain score			Mean pain score		
n=8	Pre Test	Mid test	Post Test	Pre test	Mid Test	Post test
C1	56	30	9	7	3.75	1.125
C2	66	33	8	8.25	4.125	1
C3	63	32	16	7.87	4	2
C4	69	38	10	8.62	4.75	1.25
C5	79	43	25	9.87	5.375	3.12
C6	67	34	10	8.37	4.25	1.25
C7	56	33	13	7	4.125	1.62
C8	64	33	14	8	4.125	1.75
Total	520	276	105	65	34.5	13.12
Mean	65	34.5	13.12	8.12	4.31	1.40

Evaluation Outcome of Pain

Descriptive Statistics of within group mean pain score of control group andexperimental group on 3 occasions of measurement are

Dependent Variable- Pain

Test occasion	Control groupMean	Experimental group Mean	Mean difference	N
Pre test	8.12	7.92	0.2	8
Mid test	4.31	4.26	0.05	8
Post test	1.64	1.4	0.24	8

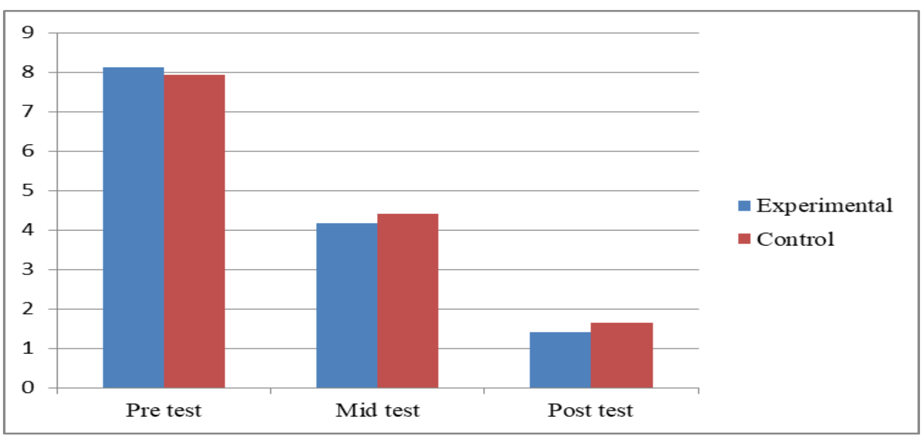


Figure 1. Mean Difference of Improvement of Pain between in Experimental and Control Group at Different Occasions.

Repeated Measures ANOVA

SV	DF	SS	MS	F	P
Treatment group	G-1	66454.083	66454.083	912.786	**.000**
Error (a)	1	1019.250	72.804	.073	.791
Occasions	2	21060.292	10530.146	615.220	**.000**
Occasions*Treatment	2	5.792	2.896	.169	.845
Error (b)	28	479.250	17.116	17.116	
Here, G=Group=2, T=Time=3, N=16					

'*' denote the significant different at 5% level

Table of repeated measured ANOVA shows that there are significant effects of booklet on pain in the experimental group. There are also significant effect on occasion on pain reduction. But There was not significant interaction between the types of treatment and occasion on the effectiveness of pain, P>0.05.

Multiple Comparisons of Means of Different Pairs of Occasion

(I) occasion	(J) occasion	Mean Difference (I-J)	SD	Sig.
Pretest	Mid test	29.875*	1.749	.000**
	Post test	51.063*	1.427	.000**
Mid test	Pretest	-29.875*	1.749	.000**
	Post test	21.188*	1.149	.000**
Posttest	Pretest	-51.063*	1.427	.000**
	Mid test	-21.188*	1.149	.000**

Table shows multiple comparison showed that the mean effect of pain in all pairs of occasions (Pretest and mid-test, pretest and mid-test, mid-test and posttest) are found to be statistically significant P<0.01.

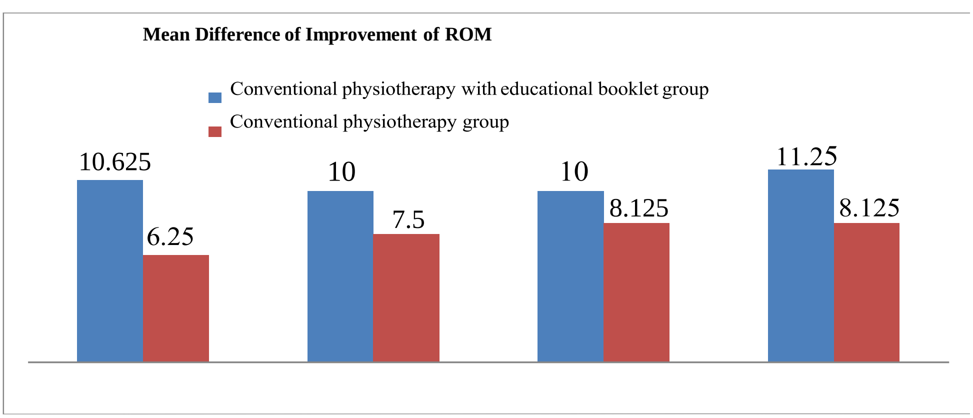
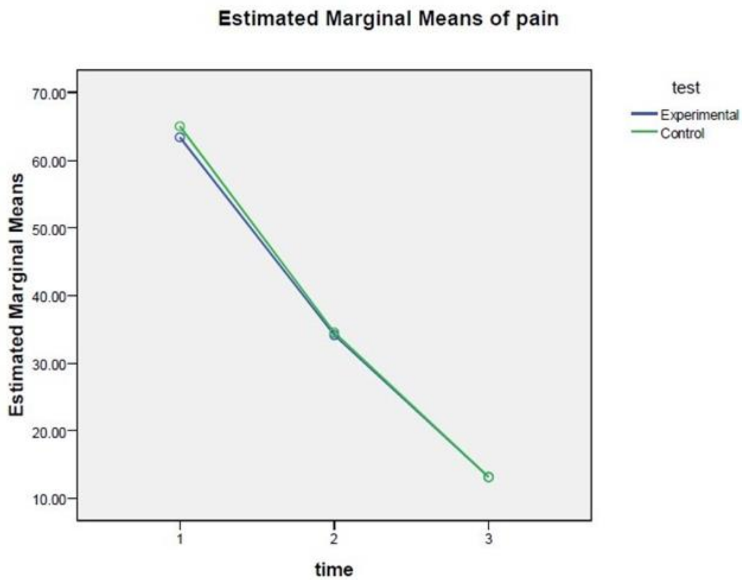


Figure 2. Mean Difference of Improvement of ROM between Pre-Test and Post-Test in Experimental and Control Group.

Line diagram showing the effect of health literacy (booklet) intervention on pain on 3different occasions.



1=Pretest, 2=Mid test and 3=Post test

The line diagram illustrates that the trend of pain in three different times have been projected downward. The tendency of pain reduction is higher in the experimental group which is substantial improvement comparing between the groups.

Improvement of ROM

Mean difference of Improvement of Range of motion between pre-test and post-test in conventional physiotherapy with educational booklet and only conventional Physiotherapy group.

Name of the variables*	Conventional physiotherapy with Educational booklet (Experimental group) (Pre test- post test)	Only conventional Physiotherapy (Control group) (Pre test- post test)
Passive flexion	10.625	6.25
Passive extension	10	7.5
Passive rotation right	10	8.125
Passive rotation left	11.25	8.125

*Rang of Motion, ROM

Table. Mean difference of Improvement of ROM between pre-test and post-test in experimental and control group

The result of this experimental study have identified that there are statistically significant difference between educational booklet with conventional physiotherapy group and only conventional physiotherapy group. That indicated that educational booklet with conventional physiotherapy is more effective than the conventional physiotherapy alone. This study demonstrated that, there are gross improvement in mean pain score and ROMin different body posture and movement.

These findings also justify interventional book's efficacy in improving health status in subjects with neck pain with the study design including internal controls to minimize bias issues and a wider range of outcomes, including multiple measures of pain, function, disability, patient satisfaction, utilization of health care services and psychosocial measures. Educational booklet is used along with conventional physiotherapy that aims to reduce pain, increase functional activity and also increase range of motion of neck, to facilitate rehabilitation program. It is helpful for better understanding of usual advice.

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Demographic Changes in Bangladesh, 1982–2020: Evidences from SVRS

A K M Ashraful Haque*
Director, Bangladesh Bureau of Statistics

M. Nurul Islam
World University of Bangladesh

Abstract

The paper reviews the demographic changes in Bangladesh from 1982 through 2020 based on the results of SVRS undertaken by BBS. A single stage stratified cluster sampling design is being followed in the system. The data reveal a substantial change in the age structure both under 15 years and over 65 years. The sex ratio showed no marked variation since 2013 remaining in the neighborhood of 105 but it declined substantially since then to little over 100.0. The dependency ratio showed substantial change from 78% in 2005 to 5.0% in 2020. Literacy rates (both 7+ and 15+ years) have also demonstrated similar marked increase: from about 53 percent in 2005 to about 75 percent in 2020. CBR fell from 34.8 in 1982 to 18.1 per thousand population in 2020. All other fertility parameters showed similar decline. Since 2009 the TFR fell to 2.04 in 2020 from 2.15 in 2009. CDR has declined from 12.2 to 4.8 per thousand population over the period 1982-2020. Interestingly IMR, NMR, PNMR, UFMR and CMR all have shown reduction of about the same magnitude remaining in the neighborhood of 60% change on the average. Maternal mortality ratio also demonstrated significant reduction: from 6.48 in 1986 to 1.63 in 2020 per 1000 life births. Life expectancy now stands at 71.2 years for males and 74.5 years for females, which were 55.3 years and 54.5 years respectively in 1981. Mean age at marriage for both males and females virtually remain static since 2013: 24.2 years for males and 18.7 years for females. CPR also remained stagnant: 62.4% to 63.9% over the last 8 years.

1. Background

Bangladesh Bureau of Statistics (BBS) introduced the Sample Vital Registration System (SVRS) for the first time in 1980 to determine the population change during the intercensal periods. Initially, its coverage was 103 primary sampling units (PSU) each consisting of 250 households. Subsequently, the number of sample PSUs was raised to 210 in 1983, 500 PSUs in 1995 and further to 1000 in 2002. To meet the data need of the planners and policymakers, the number of PSUs was further increased to 1500 in 2013. An Integrated Multi-Purpose Sample (IMPS) Design, introduced in 2012 has also been followed since 2013 SVRS. As many as 11 data recording schedules are currently being used to collect data on household and population characteristics, birth, death, migration, marriage, disability, knowledge/perception about HIV/AIDS and contraceptive use.

* Corresponding author: ahaque_62@yahoo.com

The vital events in the sample area are collected through a dual recording system known as Chandra–Deming technique proposed by Chandrasekaran and Deming. Under this system, vital events are collected as and when they occur by a locally recruited female registrar termed as Local Registrar (System 1). On the other hand, under a second system (System 2), another group of officials from District/Upazila Statistical Office of BBS also collect the data independently from the same area on quarterly basis employing four schedules bearing numbers 3 (Birth), 4 (Death), 5 (Marriage), and 6 (Divorce/Separation) and half-yearly basis employing schedules 7 (Out-Migration) and schedules 8 (In-Migration). Having the filled-in questionnaires from the two systems, data are matched in the headquarters by a pre-designed matching criteria and the demographic rates and ratios are estimated following Chandrasekaran and Deming procedure. In order to find denominators for the demographic parameters, a detailed household survey is conducted at the beginning of every year covering basic household and population characteristics. The primary objective of the present paper is to shed light on the changes in demographic behavior of the Bangladesh population over a period of over 40 years employing the data collected in SVRS area by Bangladesh Bureau of Statistics. The following areas of will be covered to study the changed referred to above:

(a) Population composition and household characteristics
(b) Fertility
(c) Mortality
(d) Marriage and marriage dissolution
(e) Migration, and
(f) Contraceptive usage

2. Coverage of the Sample

The IMPS frame developed from the 2011 census served as the sampling frame for the collection of data in the SVRS survey 2020. The master sample PSUs were used as the PSUs in the SVRS. A single–stage stratified cluster sampling methodology was adopted for the SVRS sample EAs. Prior to the selection, each of all EAs containing less than 40 households were combined with an adjacent EA to be comparable with the remaining EAs. Selection of EAs within the strata was performed with probability proportion to the estimated number of households from a computerized list ordered alphabetically within the 64 districts. Once an EA was selected, all households within the EAs were brought under the purview of data collection for SVRS area. Each of the eight administrative divisions of the country was regarded as a domain of the study. These domains were divided into three residential categories, viz. rural, urban and City Corporation. Altogether, 21 domains were thus resulted in the design. In determining the sample size for each domain, standard formulas were adopted resulting in 2012 PSUs. In 2017, 2018 and 2019 round of surveys, a total of 935 urban EAs and 1077 rural EAs were selected from the entire sample area comprising of 2012 PSUs.

3. Trends in Population Composition and Household Characteristics: 2005–2020

Table 1 presents an overview of the trends in some selected characteristics of the population and households in the SVRS area for the available years. These include, among others, age structure, dependency ratio, child-woman ratio, religious composition, literacy, household size, marital status and the like. The detailed data have been displayed in Table 1 for the period 2005–2020.

3.1 Age Structure

As reported in the SVRS, the population composition has shown a modest change since the initiation of the registration of vital events in the sample area in 2002. For example, while the population size under 15 years of age was reported to be 37.6 percent in 2005, the proportion reduced to 28.8 percent in 2018 and further to 28.1 percent in 2020. By the time, an increase was noted in the age structure at 65 years and over, from 4.2 percent in 2005 to 5.3 percent in 2020 (see Panel 1, Table 1). A similar feature of change may also be noted in the census record, from 4.0 in 2001 to 4.7 in 2011.

3.2 Sex Ratio

As evidenced in the sample area, the overall sex ratios remained almost static from 2005 to 2012, remaining in the neighborhood 105 males against 100 females. It is only 2013 when the sex ratio began to fall from 102.6 to 100.2 in 2020. This ratio is being maintained since 2017. This trend in sex ratios is in line with the one reported in the census reports also. Over the last four censuses, the sex ratio fell from 106.4 percent in 1981 to 100.3 percent in 2011. The trends in sex ratios as obtained in SVRS are shown in Table 1 (Panel 1) and Figure 1

3.3 Dependency Ratio

Dependency ratio as recorded in the SVRS, demonstrated a precipitous and continuous fall from 78 percent in 2005 to 50 percent in 2020, about 36 percent decline during 2005–2020. The 2020 survey found a ratio of the same magnitude. The census population however records this fall in the neighborhood of 7 percent, from 73 percent in 2001 to 68.4 percent in 2011. The trends in dependency ratio are shown in Figure 2 and Panel 3 of Table 1.

3.4 Religious Composition

For many years in the past, the Bangladeshi people are predominantly Muslims. Since the initiation of the SVRS program, 89.6 percent of the populations were Muslims and this proportion remained almost unchanged (89.5%) till 2010. For the last five years (2016–2020), the proportion remains constant at 88.4 percent. The Non-Muslims constitute the remaining 11.6 percent of the total (see Panel 4 of Table 1).

3.5 Literacy Rate

The literacy rate for population aged 7 years and over increased from 52.1 percent in 2005 to 75.2 percent in 2020, amounting to an increase of more than 44 percent in 16 years. The increase in female literacy compared to male literacy was more pronounced: 49.4percent for females and 39.7 percent for males (Panel 5).

The overall adult literacy rate for population aged 15 years and over increased by 40 percent over the period 2005–2020 from 53.5 percent in 2005 to 75.6 percent in 2020. The increase in literacy rate among the females was much higher (51.1%) than that of the increase among the males (34.1%) during the same period. The literacy rates of the population are shown in Panel 6.

3.6. Household Size

In line with the trends in fertility in Bangladesh, the average household size is also depicting a moderate decline over the last 15 years since 2005. As the statistics presented in Table 1 (Panel 7) show, the average size of the household in 2005 was 4.7 persons, which decreased to 4.3 in 2020: about an 11 percent decrease in the last 16 years.

Table1. Trends in Some Selected Household and Population Characteristics, SVRS 2005–20

Background Characteristics	Year															
	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
1. Age structure:																
Under15	37.6	36.6	34.9	37.4	33.3	33.1	31.9	31.1	32.3	31.7	30.8	30.8	29.3	28.8	28.5	28.1
15–64	58.2	59.3	61.0	57.9	62.3	62.4	63.5	64.2	63.2	63.5	64.6	64.6	65.6	66.2	66.2	66.6
65 & over	4.2	4.2	4.1	4.7	4.4	4.5	4.6	4.7	4.5	4.7	4.6	4.6	5.1	5.0	5.3	5.3
2. Sex ratio	105.0	105.0	105.2	105.0	104.9	105.0	104.9	104.9	102.6	100.5	100.3	100.3	100.2	100.2	100.2	100.2
3. Dependency ratio	78	76	70	67	66	65	57	56	58	57	55	54	53	51	51	50
4. Religion:																
Muslim	89.3	89.3	89.4	89.4	89.4	89.5	88.8	88.8	89.1	89.2	88.2	88.4	88.4	88.4	88.4	88.4
Non-Muslim	10.7	10.7	10.6	10.6	10.6	10.5	11.2	11.2	10.9	10.8	11.8	11.6	11.6	11.6	11.6	11.6
5. Literacy 7+:																
Both sexes	52.1	52.5	56.1	55.8	56.7	56.8	55.8	56.3	57.2	58.6	63.6	71.0	72.3	73.2	74.4	75.2
Male	55.4	55.8	59.4	60.8	59.6	59.8	58.4	59.2	59.3	60.7	65.6	73.0	74.3	75.2	76.5	77.4
Female	48.8	49.1	52.7	52.7	53.8	53.9	53.2	53.3	55.1	56.6	61.6	68.9	70.2	71.2	72.3	72.9
6. Literacy15+:																
Both sexes	53.5	53.7	56.3	56.9	58.4	58.6	58.8	60.7	61.0	61.4	64.6	72.3	72.9	73.9	74.7	75.6
Male	58.3	58.5	63.1	61.3	62.6	62.9	62.5	64.8	64.2	64.7	67.6	75.2	75.7	76.7	77.4	78.2
Female	48.6	48.8	53.5	52.6	54.3	55.4	55.1	56.6	51.8	58.2	61.6	69.5	70.1	71.2	71.9	73.0
7. Household size	4.7	4.8	4.7	4.7	4.7	4.6	4.5	4.5	4.4	4.3	4.4	4.3	4.2	4.2	4.2	4.3

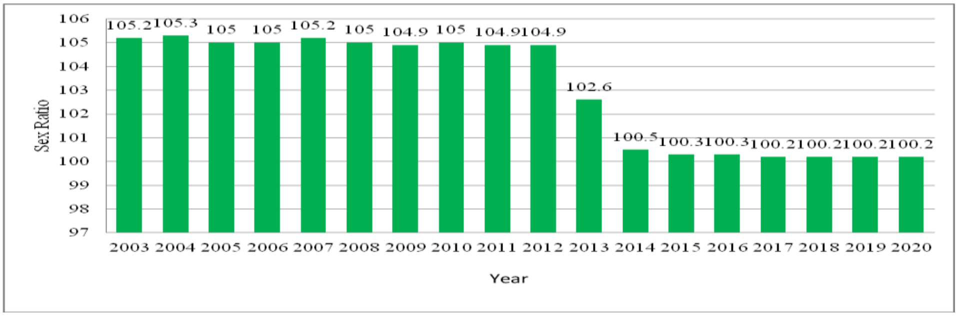

Figure. Trends in Sex Ratios, SVRS 2003-2020

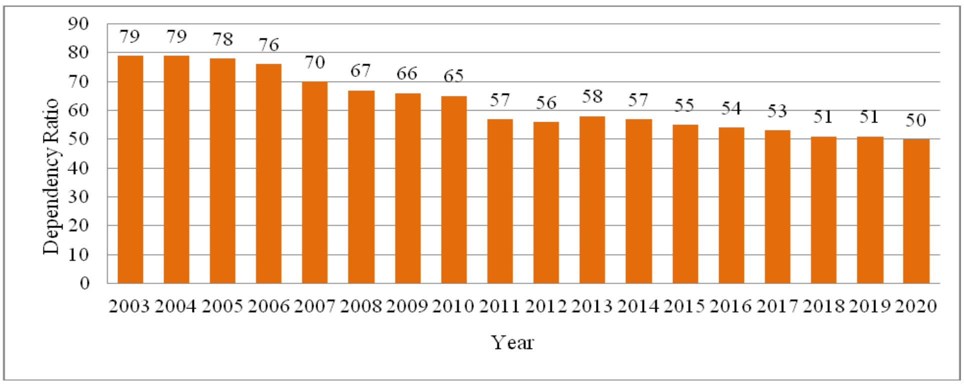

Figure 2. Trends in Dependency Ratios, SVRS 2003-20

4. Trends in Fertility and Reproduction: 1982-2020

The trends in fertility over time have been examined in this section by comparing the CBR, GFR, TFR, GRR and NRR for the overall sample since 1982. Table 2 presents these estimates. The crude birth rate remained in the neighborhood of 35 till 1986, which thereafter began to decline and reached to 18.9 in 2001, implying almost a 54 percent fall in about 15 years. The rate then recorded a slow rise for a short period of about 2 to 3 years and then started again to decline reaching to its lowest level (18.1) as recorded in the last SVRS undertaken in 2020. The GFR also displays the same characteristic features. Beginning with a value of as high as 164 in 1982, the rate reached to 65 in 2020 implying over 60 percent decline in 38 years. The TFR declined sharply from 5.21 births per woman in 1982 to 2.04 in 2020. As the data show, the TFR has possibly reached a plateau in recent time with a value in the neighborhood of 2.1. The GRR and NRR demonstrate the same feature of trends as discerned by the remaining measures of fertility. Available measures of fertility and reproduction tend to suggest that Bangladesh has possibly reached nearly to scenario of replacement level of fertility. The fertility rates in questions have been presented in Figure 3.

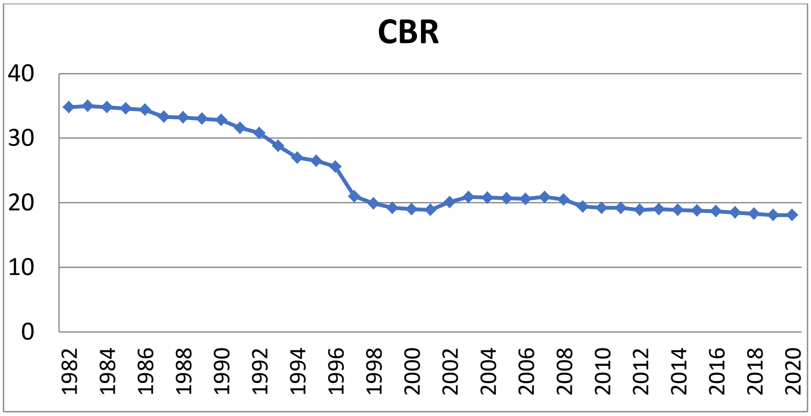

Figure 3.A. Trends in CBR as Observed in the SVRS Area, 1982-2020

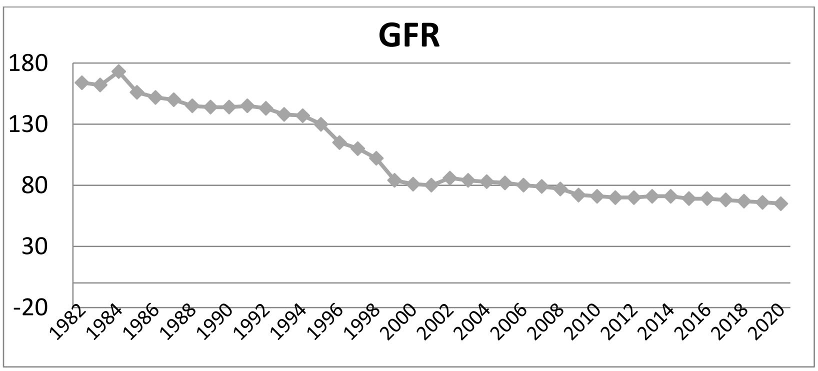

Figure 3.B. Trends in GFR as Observed in the SVRS Area, 1982-2020

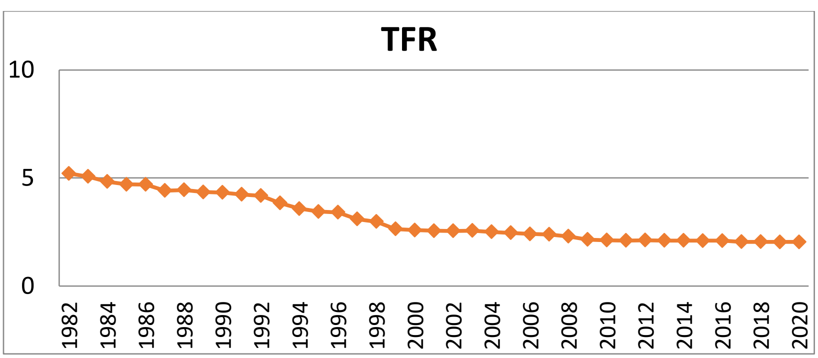

Figure 3.C. Trends in TFR as Observed in the SVRS Area, 1982-2020

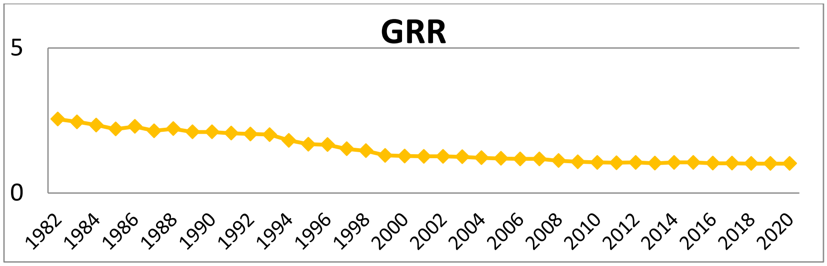

Figure 3.D. Trends in GRR as Observed in the SVRS Area, 1982-2020

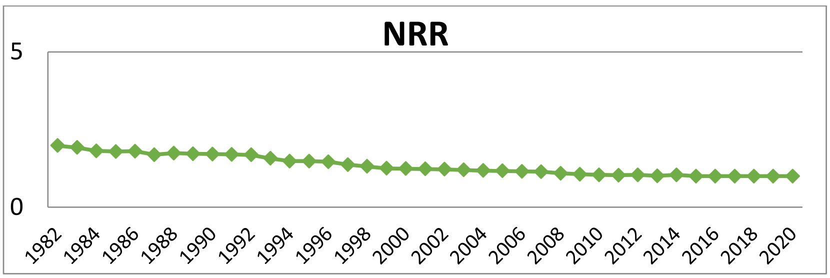

Figure 3.E. Trends in NRR as Observed in the SVRS Area, 1982-2020

5. Trends in Mortality: 1982-2020

5.1 Crude Death Rate

The crude death rates estimated by BBS through its SVRS program are presented in Table 3 since 1982. The rate was in the neighborhood of 12 per thousand populations during 1982–95, which thereafter declined to 10 per thousand in 1993. However, the onset of a fast decline in the level of crude death rate was observed in 1994 which recorded a further decline to 5.1 in 2002. A temporary rise in the CDR was noted after this period. The current CDR is estimated to be 5.1 per thousand populations. The rates from 2002 onward are the ones derived from the registered deaths in the SVRS area of BBS.

Table 2. Trends in Crude death Rates for Bangladesh, SVRS 1982-2020

Year	Crude death rate	Period	Crude death rate
1982	12.2	2001	4.8
1983	12.3	2002	5.1
1984	12.3	2003	5.9
1985	12.0	2004	5.8
1986	12.1	2005	5.8
1987	11.5	2006	5.6
1988	11.3	2007	6.2
1989	11.3	2008	6.0
1990	11.4	2009	5.8

Year	Crude death rate	Period	Crude death rate
1991	11.2	2010	5.6
1992	11.0	2011	5.5
1993	10.0	2012	5.3
1994	9.3	2013	5.3
1995	8.7	2014	5.2
1996	8.2	2015	5.1
1997	5.5	2016	5.1
1998	5.1	2017	5.1
1999	5.1	2018	5.0
2000	4.9	2019 2020	4.9 5.1

Sources: (1) For the period 1881–1980: CPD–UNFPA Paper Series,
(2) For 1981–2011, BBS (2013, 2014), (3) *SVRS–2011 Key Indicators (BBS, 2020)

5.2 Childhood Mortality

As the data in Table 4 display, neo-natal mortality, under-five mortality and childhood mortality rates all have declined consistently from 2001 to 2020. Even more impressive is the decline in under-five mortality and post-neonatal mortality, which showed 65.8 percent and 64.7 percent decline over the period under study. Infant mortality, neo-natal mortality and child mortality showed a decline of 62.5 percent, 61.5 percent and 58.5 percent respectively, each over the same period.

Table 3. Trends in Childhood Mortality Rates, SVRS 2001-2020

Year	Infant mortality	Neonatal mortality	Post-neonatal mortality	Under-five mortality	Child mortality
2001	56	39	17	82	4.1
2002	53	36	17	76	4.6
2003	53	36	17	78	4.6
2004	52	36	17	74	4.5
2005	50	33	16	68	4.1
2006	45	31	14	62	3.9
2007	43	29	13	60	3.6
2008	41	31	10	54	3.1
2009	39	28	11	50	2.7
2010	36	26	10	47	2.6
2011	35	23	11	44	2.4
2012	33	22	12	42	2.3
2013	32	22	11	41	2.2
2014	30	21	09	38	2.0
2015	29	20	09	36	2.0

Year	Infant mortality	Neonatal mortality	Post-neonatal mortality	Under-five mortality	Child mortality
2016	28	19	09	35	1.8
2017	24	17	07	31	1.8
2018	22	16	06	29	1.7
2019 2020	21 21	15 15	06 06	28 28	1.7 1.7
% change (2001–2019)	**62.5**	**61.5**	**64.7**	**65.8**	**58.5**

Sources: BBS (2014), SVRS–2013 Key Indicators (BBS, 2015), na: Not available

5.3 Maternal Mortality Ratio

The trends in MMR during the period 1986–2020 are shown in the accompanying table (Table 5). As the estimates presented in the table dictate, the MMR declined from 6.48 per 1000 live births in 1986 to 3.15 in 2001, a more than 51 percent decline in 15 years. The vital registration system initiated in 2002 records a somewhat higher rate (3.93) compared to the previous year's obtained from other sources. This ratio falls consistently to 1.63 in 20120 from its 1986 level of an MMR of 64.8, a decline of 74.8 percent over a period of 35 years. Figure 3 shows the trends in maternal mortality ratios over the period 2002–2020.

Table 4. Trends in Maternal Mortality Ratio Per 1000 Live Births, SVRS 1986–2020

Year	MMR	Year	MMR
1986	6.48	2003	3.76
1987	5.96	2004	3.65
1988	5.72	2005	3.48
1989	5.08	2006	3.37
1990	4.78	2007	3.51
1991	4.72	2008	3.48
1992	4.68	2009	2.59
1993	4.52	2010	2.16
1994	4.49	2.011	2.09
1995	4.47	2012	2.03
1996	4.44	2013	1.97
1997	3.50	2014	1.93
1999	3.20	2015	1.81
2000	3.18	2016	1.78
2001	3.15	2017	1.72
2002	3.91	2018	1.69
2002	3.91	2019 2020	1.65 1.63
% change in MMR (1987–2019):			**55.3**

Source: BBS (2013,2014), *SVRS–2013 Key Indicators (BBS, 2020)

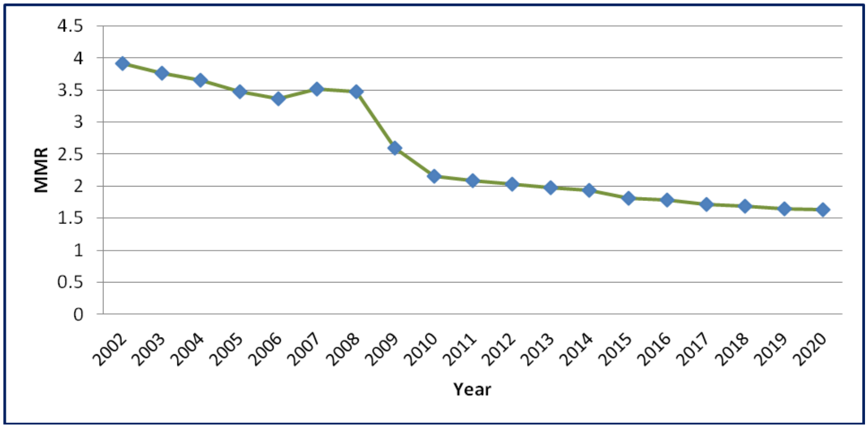

Figure 4. Trend in Maternal Mortality Ratio, SVRS 2002-2020

5.4 Expectation of Life at Birth

Expectation of life at birth is a summary measure of mortality that portrays the average longevity of life of an individual. The vital registration system in Bangladesh maintained and monitored by the Bangladesh Bureau of Statistics provides the estimates of life expectancy over the last 40 years. These estimates are shown in Table 6 for the period 1981–2020. The trends in the expectation of life at birth are displayed in Figure 4. Note that the expectations of life at birth for males and females were 55.3 and 54.5 in 1981. These increased to 71.2 and 74.5 years in 2020 over a period of 40 years.

Table 5. Trends in Expectation of Life at Birth by Sex, SVRS 1981–2020

Year	Male	Female	Year	Male	Female
1981	55.3	54.5	2001	64.0	64.5
1982	54.5	54.8	2002	64.5	65.4
1983	54.2	53.6	2003	64.3	65.4
1984	54.9	54.7	2004	64.4	65.7
1985	55.7	54.6	2005	64.4	65.8
1986	55.2	55.3	2006	65.4	67.8
1987	56.9	56.0	2007	65.5	67.9
1988	56.5	55.6	2008	65.6	68.0
1989	56.0	55.6	2009	66.1	68.7
1990	56.6	55.6	2010	66.6	68.8
1991	56.5	55.7	2011	67.9	70.3
1992	56.8	55.9	2012	68.2	70.7
1993	58.2	57.7	2013	68.8	71.2
1994	58.2	57.9	2014	69.1	71.6
1995	58.4	58.1	2015	69.4	72.0

Year	Male	Female	Year	Male	Female
1996	59.1	58.6	2016	70.3	72.9
1997	60.3	59.7	2017	70.6	73.5
1998	61.7	61.2	2018	70.8	73.8
1999	63.0	62.4	2019	71.1	74.2
2000	63.7	63.5	2020	71.2	74.5

Source: BBS (2014), *SVRS–2013 Key Indicators (BBS, 2020)

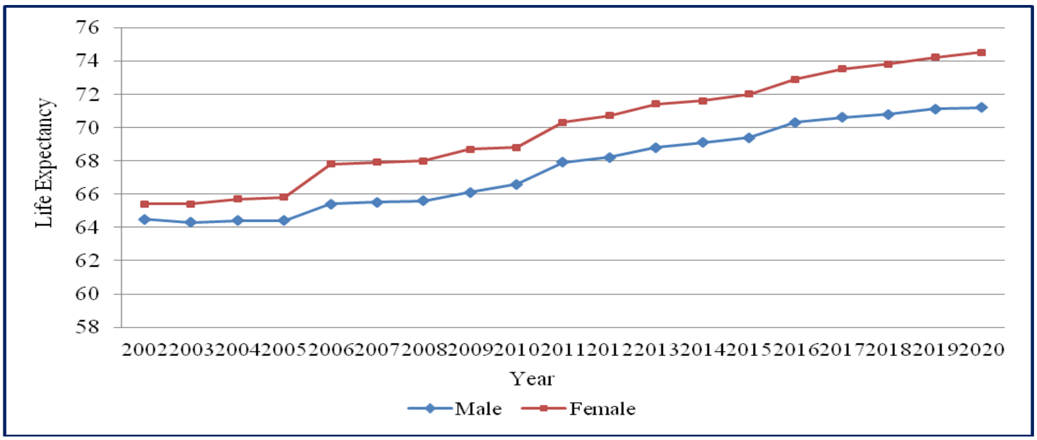

Figure 5. Trends in Expectation of Life at Birth by Sex, SVRS 2002–2020

6. Trends in Marriage, Divorce and Separation: 2005-2020

The trends in some marriage and marriage related indicators are summarized in Table 7. The crude marriage rate shows a substantial increase over the last 16 years, from 13.0 per thousand populations in 2005 to 15.3 per thousand populations in 2020, an increase of about 18 percent over the stated period. The increase in general marriage rates for both males and females have been pronounced during 2005-2020: from 19 in 2005 to 4.5 in 2020 for males. The corresponding rates for females are 21.5 and 42.4.

There has been virtually no change in the crude separation rate over the period under investigation. The Singulate mean age at marriage (SMAM) for both males and females has marked a negligible and irregular increase during this period. There is a tendency for the crude divorce rate to increase over time: from 0.7 in 2005 to 1.0 in 2020, although the pattern of increase is somewhat erratic. The mean age at first marriage remains static over the last nine years or so.

Before we conclude, it may be worth to mention that the measures of marriage and marital dissolution presented in this chapter tend to reflect that there have been virtually very little changes in these measures during the last 15 years or so. In some cases, the levels and patterns of these measures are erratic and irregular. It may thus be very difficult to bring favorable changes unless concerted efforts from all walks of life are strengthened.

7. Trends in Contraceptive Use: 2005-2020

There has been a gradual increase in the use of contraceptive methods in Bangladesh over the last 45 years since 1975 when the First Bangladesh Fertility Survey was undertaken recording a contraceptive prevalence rate of 7.7 percent. The Bangladesh Demographic and Health Survey (BDHS) of 2014 reported this rate to be 62.4 percent, a more than 8-fold increase over this period. The SVRS area also demonstrated a substantial increase from 57.0 in 2005 to 62.5 in 2017, nearly a 10 percent increase in about 13 years' time. During this period, the increase in the contraceptive use rate in rural area was also about 14.3 percent, from 55.2 percent in 2005 to 63.9 percent in 2020. Table 8 presents an overview of the trends in contraceptive use since the initiation of the SVRS program of registration of the vital events in Bangladesh.

Note that, while the modern method use has shown an increase of more than 20 percent during 2005–2020, the traditional method use has correspondingly gone down by about 75 percent. Use of condom over this time recorded an erratic increase from 5.2 percent in 2005 to 6.6 percent in 2020, while the use of oral pill remained almost static remaining somewhere in the neighborhood of 35 percent reaching at 36.8 percent in 2020.

Table 6. Trends in Indicators of Marriage, Divorce and Separation, SVRS 2005-2020

Background Characteristics	Year															
	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
Crude marriage rate:	13.0	12.4	12.5	11.6	13.2	12.7	13.4	13.3	13.0	12.9	13.0	14.3	14.6	14.7	14.9	15.3
General marriage rate:	20.5	19.6	19.2	17.4	19.6	18.4	19.7	19.3	19.1	19.0	18.8	20.6	20.7	20.6	20.8	21.2
Male	19.0	18.3	18.2	16.1	18.1	17.4	18.1	38.1	38.1	38.1	37.9	41.3	41.4	41.4	41.7	42.5
Female	21.5	21.0	20.1	18.8	21.1	20.3	21.2	39.1	38.4	37.7	37.4	41.2	41.3	41.0	41.5	42.4
Crude divorce rate:	0.7	0.6	0.6	0.6	0.7	0.8	0.8	0.8	0.6	.09	0.9	1.1	1.0	0.9	1.0	1.0
General divorce rate:																
Male	–	0.5	–	–	–	–	–	0.7	1.8	2.8	2.6	3.1	2.8	2.6	2.7	2.7
Female	–	1.6	–	–	–	–	–	1.7	0.9	2.7	2.6	3.1	2.8	2.6	2.7	2.7
Crude separation rate:	0.3	0.2	0.3	0.2	0.3	0.2	0.3	0.3	0.3	0.3	0.4	0.6	0.3	0.3	0.3	0.3
General separation rate:																
Male	—	0.3	–	–	–	–	–	0.4	0.8	0.8	1.0	1.1	0.9	0.9	0.7	0.7
Female	–	0.6	–	–	–	–	–	0.6	0.8	0.8	1.0	1.1	0.9	0.9	0.7	0.7
Mean age at marriage:																
Male	25.3	23.4	23.6	23.8	23.8	23.9	24.9	24.8	24.3	25.9	26.4	26.3	26.2	25.5	25.3	25.2
Female	17.9	18.1	18.4	19.1	18.5	18.7	18.6	19.3	18.4	18.5	18.7	18.8	18.8	18.9	18.9	19.1
Median age at marriage:																
Male	–	–	–	–	–		24.0	25.0	24.0	24.0	25.0	25.0	25.0	24.0	24.0	24.0
Female	–	–	–	–	–	–	18.0	19.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0
Mean age at first marriage:																
Male	–	–	–	–	–	–	–	–	24.3	24.9	25.3	25.2	25.1	24.4	24.2	24.2
Female	–	–	–	–	–	–	–	–	17.9	18.3	18.4	18.4	18.4	18.6	18.5	18.7
Median age at first marriage:																
Male	–	–	–	–	–	–	–	–	24.0	24.0	25.0	25.0	25.0	24.0	24.0	24.0
Female	–	–	–	–	–	–	–	–	18.0	18.0	18.0	18.0	18	18.0	18.0	18.0
Singulate mean age at marriage (SMAM):																
Male	25.6	25.7	25.6	25.9	26.0	26.1	26.1	26.0	25.47	25.4	25.8	25.7	25.6	26.0	26.3	26.1
Female	19.5	19.3	19.4	20.3	20.3	20.2	20.5	20.3	20.02	20.0	20.3	20.3	20.3	20.7	20.5	20.8

(–): Not available

Table 7. Trends in Current use of Contraceptive Methods (%), SVRS 2005–2020

Method	Years															
	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
Any method	57.0	58.3	55.0	52.6	56.1	56.7	58.3	62.2	62.4	62.2	62.1	62.3	62.5	63.1	63.4	63.9
Any method (rural	55.2	57.1	53.8	51.1	54.4	55.3	56.0	59.8	61.1	61.6	60.4	59.3	59.4	62.4	62.7	63.1
Any method (urban)	60.4	60.5	57.0	55.3	58.7	60.9	62.2	66.1	63.4	64.5	64.5	65.9	66.3	64.0	64.4	64.7
Any modern method:	**51.7**	**52.5**	**51.8**	**50.6**	**53.6**	**54.8**	**56.6**	**60.2**	**60.0**	**58.4**	**58.4**	**58.4**	**59.2**	**61.6**	**62.2**	**63.5**
Condom	5.2	6.8	4.4	3.2	5.5	3.8	4.0	5.3	5.0	5.1	7.2	5.8	8.6	7.2	8.0	6.6
Oral pill	35.4	36.2	34.5	37.9	37.1	34.4	35.0	35.8	36.1	34.8	32.7	33.4	33.4	34.9	35.6	36.8
Injections	8.5	7.0	10.3	8.0	9.0	12.7	12.8	14.0	14.6	14.7	14.5	15.2	13.4	15.3	14.4	14.7
Male sterilization	0.2	0.3	0.3	0.2	0.2	0.4	0.5	0.49	0.6	0.5	0.3	0.3	0.3	0.3	0.3	0.4
Copper–T	0.6	0.7	0.8	0.4	0.4	0.8	0.9	1.1	0.9	0.9	1.0	0.8	0.9	1.0	1.0	1.1
Female sterilization:	1.8	1.7	1.9	0.9	1.3	2.0	2.1	2.5	1.8	1.7	1.8	2.0	1.6	1.8	1.7	1.7
Foam	NA	NA	NA	NA	NA	NA	0.4	0.6	0.5	0.4	0.3	0.4	0.4	0.5	0.5	0.5
Norplant	NA	NA	NA	NA	NA	0.0	0.5	0.6	0.6	0.5	0.5	0.5	0.5	0.5	0.1	0.5
Any traditional method	5.1	5.3	5.8	3.2	2.1	2.5	2.0	1.8	2.0	2.4	3.8	3.9	3.3	1.5	1.3	1.6

NA: Not Available

Trends in CPR by locality in case of current use are provided in Figure 6.

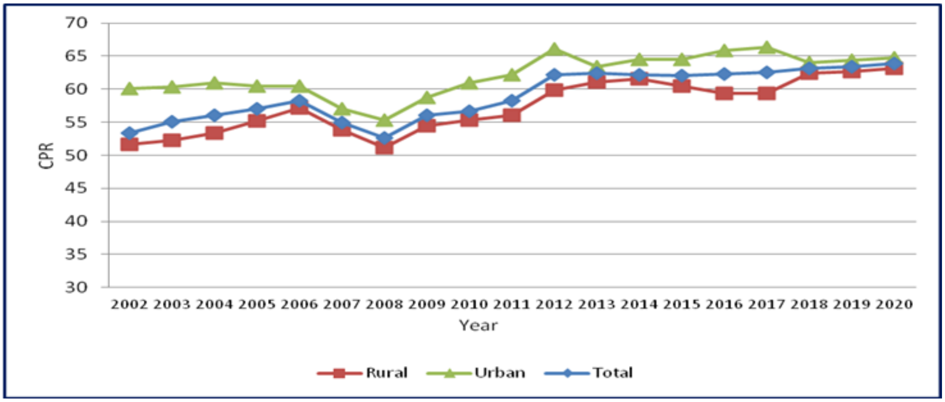

Figure 6. Trends in Current use of Contraception by Locality, SVRS 2002-2020

8. Trends Internal Migration

The trends in migration rates in Bangladesh over the last 30 years both in and out are shown in Figure 6 and Figure 7. Figure 8 shows the overall trends in out and in-migration rates for the same period. All these figures demonstrated an increasing trend in, out and net migration rates over time.
As we noted from the data (Tables not shown), the overall in-migration rate in the sample area in 2020 is 69.2 per 1000 population

The detailed tabulation showed that the overall in-migration rate as found in the sample area in 2020 is 69.2 per thousand population. This when compared with an out-migration rate of 68.8 per thousand population, results in a net gain of 0.4 persons per thousand population. These rates were 72.4 and 72.7 in 2019 resulting in a loss of 0.3 persons per 1000 population.

Migratory movement of the females was more pronounced than their male counterparts. For example, while only 60.8 per thousand males moved into the sample area, the corresponding rate for females was to the extent of 77.5 per thousand. A similar feature of movement was also noted in the case of out-migration: 60.0 for males and over 77.5 for females. It is surprising to note that for females; in and out migration balanced each other resulting in a static size of the population.

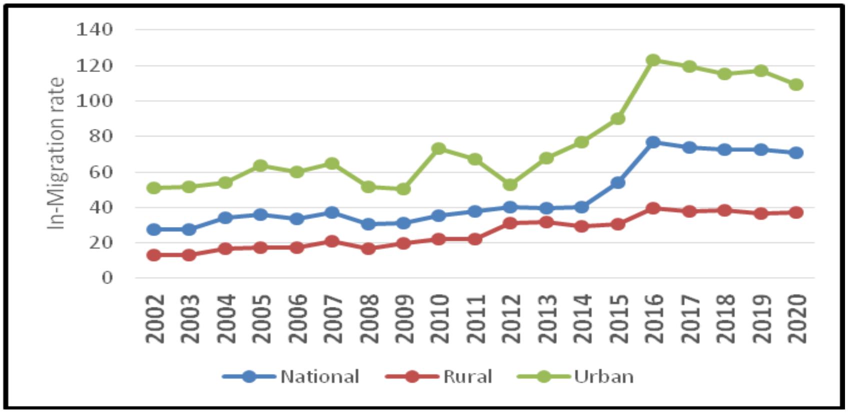

Figure 7. In-Migration Rates Per 1000 Population, SVRS 2002-2020

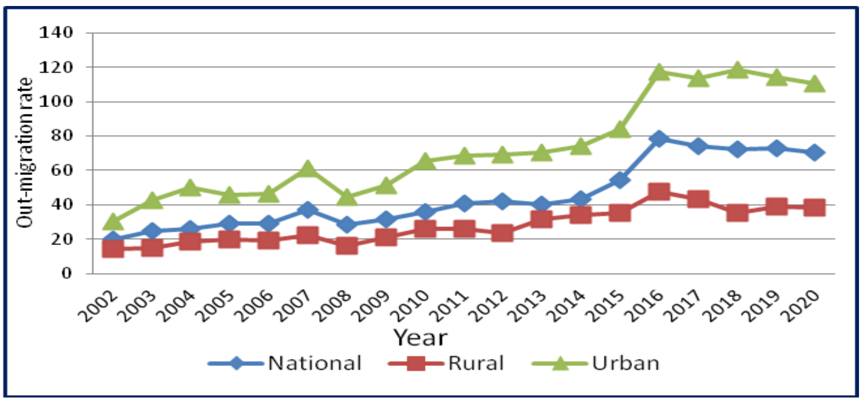

Figure 8. Out-Migration Rates Per 1000 Population, SVRS 2002-2020

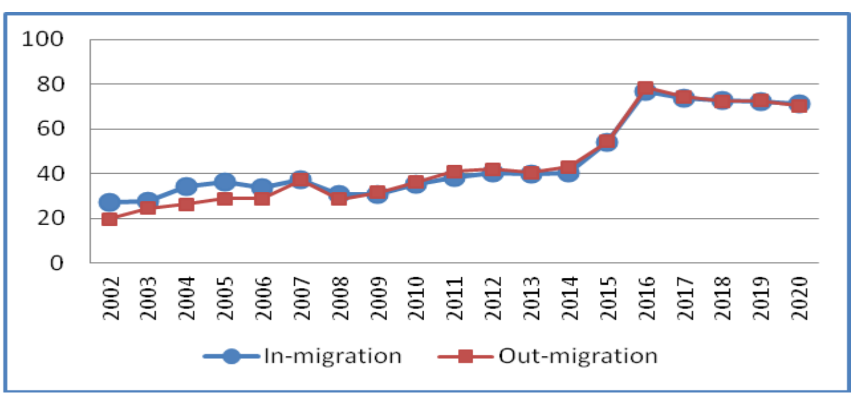

Figure 9. In-Migration & Out-Migration Rates Per 1000 Population, SVRS 2002-2020

9. Discussion and Conclusions

The onset of fertility decline as recorded in SVRS and in other documents was evident since the mid-seventies. The decline occurred at a rapid pace during the period 1975 to 1993-94. The total fertility rate was 3.41 in 1992 and decreased to 2.04 in 2020. It is only recently, that the TFR has reached to a reasonable level indicating that we are very close to the doorstep of reaching replacement level of fertility. This has been possible perhaps because of a sharp rise in the contraceptive use rate in the recent past and more so in response to a positive attitude of the couples to limit their family size. The population program of the government also helped in achieving this decline in fertility. Mortality level is reasonably low. This has led to a substantial increase in the expectation of life. Age at marriage needs to be raised further in order to keep the replacement level of fertility sustainable. Yet the overall economic growth will be severely affected due to the population momentum, and without integrated policy measures, it would be difficult to face the emerging challenges.

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Size and Power Properties of Some Test Statistics for Testing the Process Capability Index

Shipra Banik
Department of Physical Sciences, Independent University, Bangladesh, Bashundhara, Dhaka 1229, Bangladesh

B. M. Golam Kibria*
Department of Mathematics and Statistics, Florida International University, Miami, Florida, USA

Abstract

This paper considers some test statistics, namely, the trimmed robust statistic, the adjusted degrees of freedom statistic, the large sample statistic, the augmented large sample statistic and a bootstrap version of the classical statistic for testing the population process capability index. To evaluate the performances of the test statistics, empirical sizes and powers are calculated at the 5% nominal level and compared with the classical statistic under both symmetric and the skewed distributions. It is evident from simulation study that our proposed tests have better size and power properties as compared to the classical approach.

Keyword: Hypothesis testing; Monte Carlo simulation; Power of the test; Process capability index; Symmetric and skewed distributions, Size of the test.

1. Introduction

Process capability index (PCI) determines the extent of deviation of a process relative to its specification limits. There are several statistics that can be used to measure the capability of a process. However, Cp is the most commonly used PCI (Kane (1986), Juran (1974) and Zhang (2010)). It is the fraction of the spread between the process specifications to the spread of the process values, as measured by six process standard deviation units. More clearly, it gives the size of the range over which the process actually differs. We emphasis in this paper on the PCI, Cp suggested by Kane (1986), which is defined as follows:

$$C_p = \frac{USL-LSL}{6\sigma}, \tag{1.1}$$

where USL is the upper specification limit, LSL is the lower specification limit and σ is the process standard deviation. The numerator of Cp provides the size of the range over which the process capacities can differ. The denominator offers the size of the range over which the process essentially differs (Kotz and Lovelace (1998)). If σ is unknown, it must be estimated from the sample data. The estimator of the PCI, Cp is hence,

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* Corresponding author: kibriag@fiu.edu

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6S}, \tag{1.2}$$

where S is the sample standard deviation.

Although the point estimator of Cp can be a convenient measure, however, it is evident that the interval estimate, and hypothesis test for Cp are also essential. Several researchers have considered various methods for estimating Cp by the confidence interval methods. To mention a few, Abu-Shawiesh et al. (2020 ab), Hummel and Hettmansperger (2004), Panichkitkosolkul (2014, 2016), Zhang, J. (2010) among others. However, the literature on the test statistics for testing the Cp is very limited in literature. This paper made an effort to consider and also to propose some new test statistics for testing the population Cp and compare them under the same simulation condition. It is important to note that tests with correct sizes and good powers are essential, particularly in finite samples. This paper is organized as follows. The theoretical background and an analytical formula of the existing and suggested test statistics are provided in section 2. To compare the performance of the test statistics, a simulation study has been conducted in Section 3. Some concluding remarks are outlined in Section 4.

2. Statistical Methodology

In this section, we will review and propose some new test statistics for testing the null hypothesis H_0: Cp≤1.0 (process is not capable) against the alternative hypothesis H_1: Cp>1.0 (process is a capable), is as follows:

2.1 Classical Test

Suppose $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$, then it can be shown that $\frac{(n-1)\,\hat{C}_p^2}{C_p^2}$ has a Chi-square distribution with (n-1) degrees of freedom. Then, to test H_0: Cp≤1.0 vs. H_1: Cp>1.0, the test statistic is defined as

$$\chi^2 = \frac{(n-1)(Cp)^2}{\widehat{Cp}^2} \tag{2.1}$$

where $\widehat{Cp}$ is the sample estimate of population Cp. At α level of significance, the null hypothesis will be rejected when $\chi^2 > \chi^2_{1-\alpha,n-1}$ and $\chi^2_{1-\alpha,n-1}$ is the upper 1− α quintile of the central chi-squared distribution with n-1 degrees of freedom.

2.2 Test Based on Adjusted Degrees of Freedom

Hummel and Hettmansperger (2004) proposed an estimate for the degrees of freedom using the method of matching. It depends on the fact that the sample variance is a sum of squares and, for sufficiently large samples, is approximated as a chi-square estimate with the appropriate degrees of freedom. They matched the first two moments of the distribution of sample variance with that of a random variable X, which is distributed as $c\chi^2_r$. The solution for r and c is solved using the following systems of equations:

1) σ^2 = cr and 2) $\frac{\sigma^4}{n}\left(\kappa-\frac{n-3}{n-1}\right)$=2rc^2, where κ is the kurtosis of the distribution. Following Panichkitkosolkul (2016), to test H_0: Cp≤1.0 vs. H_1: Cp>1.0, the test statistic is defined as

$$\chi^2 = \frac{\hat{r}\,(Cp)^2}{\widehat{Cp}^2} \tag{2.2}$$

where $\hat{r} = \frac{2n}{\hat{\gamma}+2n/(n-1)}$ and $\hat{\gamma} = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \frac{\sum_{i=1}^{n}(X_i-\bar{X})^4}{S^4} - \frac{3(n-1)^2}{(n-2)(n-3)}$.

At α level of significance, the null hypothesis will be rejected when $\chi^2 > \chi^2_{1-\alpha,\hat{r}}$ and $\chi^2_{1-\alpha,\hat{r}}$ is the upper 1− α quintile of the central chi-squared distribution with $\hat{r}$ degrees of freedom.

2.3 Test Based on the Large Sample Theory

If the normality assumption is invalid, then one can use the large sample theory, where $S^2 \sim N(\sigma^2, \frac{\sigma^4}{n}(\kappa_e + \frac{2n}{n-1}))$, κ_e is the excess kurtosis. Following, Panichkitkosolkul (2016), to test H_0: Cp≤1.0 vs. H_1: Cp>1.0, the test statistic is defined as

$$Z = \frac{2log\hat{c}_p - 2log c_p}{\sqrt{A}} \tag{2.3}$$

where A = $\frac{G_{2+2n/(n-1)}}{n}$ and $G_2 = \frac{n-1}{(n-2)(n-3)}[(n-1)g_2+6]$, $g_2 = \frac{m_4}{m_2^2} - 3$, $m_4 = n^{-1}\sum_{i=1}^{n}(X_i - \overline{X})^4$ and $m_2 = n^{-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$. For critical value of the test statistic, see the standard Z table.

2.4 Test Based on the Augmented Large Sample Theory

Burch (2014) considered a modification to the approximate distribution of log(S) by using a three-term Taylor's series expansion. Employing the large sample properties of S^2, and following Burch (2014) and Panichkitkosolkul (2016), to test H_0: Cp≥1.0 vs. H_1: Cp<1.0, the test statistic is defined as

$$Z = \frac{2log\ \hat{c}_p - 2log c_p}{\sqrt{B}}, \tag{2.4}$$

where B= $\widehat{var}\log(S^2) \approx \frac{1}{n}\left(\kappa_e + \frac{2n}{n-1}\right)\left(1 + \frac{1}{2n}\left(\kappa_e + \frac{2n}{n-1}\right)\right)$, C = $\frac{\hat{\kappa}_{e,5}+2n/(n-1)}{2n}$, $\hat{\kappa}_{e,5} = \left(\frac{n+1}{n-1}\right)G_2(1 + \frac{5G_2}{n})$. For critical value of the test statistic, see the standard Z table.

2.5 The Proposed Robust Test

The sample mean and the sample variation can be influenced by the outliers or extreme values of the distribution. To overcome the extreme value problem, the trimmed technique is very useful (Burch (2014), Tukey (1948) and Dixon and Yuen (1974) among others). To modify the variance of the trimmed mean, Sindhumol et al. (2016) recommended an amendment, which is multiplying the variance of the trimmed mean with a fine-tuning constant. This technique can be described as follows: Consider $X_i \sim N(\mu,\sigma^2)$, i=1,2,…,n. Assume that the order statistics of above random samples is denoted by $X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)}$. Then the r-times symmetrically trimmed sample is obtained by reducing both bottommost and uppermost r values. Then the trimmed sample mean and the trimmed sample standard deviation is defined respectively as follows: $\bar{X}_T = \frac{1}{n-2r}\sum_{i=r+1}^{n-r} X_{(i)}$ and $S_T = \sqrt{\frac{1}{n-2r-1}\sum_{i=r+1}^{n-r}(X_{(i)} - \bar{X}_T)^2}$, where $r = [\alpha n]$, trimming is done for α% ($0 \le \alpha \le 0.5$) of n. The modified trimmed standard deviation, suggested by Sindhumol et al. (2016)) and is defined as follows: $S^*_T = 1.4826 S_T$ (For details, see Abu-Shawiesh et al. (2020a, b)). Now, in the view of equations (2.1) and (2.2), to test H_0: Cp≤1.0 vs. H_1: Cp>1.0, the test statistic is defined as

$$\chi^2 = \frac{(n-1)(Cp)^2}{\hat{C}_p^{*2}} \tag{2.5}$$

where $\hat{C}_p^* = \frac{\text{USL} - \text{LSL}}{6\, S_T^*}$ is the sample estimate of population Cp. For critical value of the statistic, see the standard Chi-square table.

2.6 Bootstrap Test

Bootstrap is a frequently used non-parametric approach (introduced by Efron (1979)), which involves no assumptions about the primary population and can be applied to a range of situations. The accurateness of the bootstrap statistic relies on the number of bootstrap samples. If the number of bootstrap samples is large enough, the estimate may be precise. A bootstrap method is summarized as follows: Let $X^{(*)} = X_1^{(*)}, X_2^{(*)}, \ldots, X_n^{(*)}$, where the i^{th} sample is denoted $X^{(i)}$ for i=1,2 ,…, B, and B are the number of bootstrap samples. The number of bootstrap samples is naturally between 1000 and 2000. Following, Panichkitkosolkul (2014) to test H_0: Cp≤1.0 vs. H_1: Cp>1.0, the test statistic is defined as

$$t_0^* = \frac{1}{2}\left(\frac{Cp^2}{\widehat{Cp}^2} S^2 k_1 - k_1\right), \tag{2.6}$$

where $k_1 = \sqrt{2n-2}$. We will reject the null hypothesis, when $t_0^* > \hat{t}_{1-\alpha}^*$, where $\hat{t}_{1-\alpha}^*$ is the quintiles of the following statistic, $T^* = \frac{S^{*^2} - S^2}{\sqrt{\widehat{var}\left(S^{*^2}\right)}}$, where S^{*^2} is a bootstrap replication of the statistic S^2, $\widehat{var}\left(S^{*^2}\right) = \frac{1}{n}\left(\hat{\mu}_4^* - \frac{n-3}{n-1} S^{*^4}\right)$and $\hat{\mu}_4^* = \frac{1}{m}\sum_{i=1}^{m}(X_i^* - \bar{X}^*)^4$.

3. Simulation Study

3.1 Simulation Design

The main objective of this paper is to find some good statistics for testing the population Cp. Since a theoretical judgement is not likely, a simulation study has been made to compare the size and the power performances of the considered test statistics for the following distributions, which contains both symmetric and skewed:

(i) Standard Normal distribution, N(50,1)
(ii) Chi-Square distribution, $\chi^2_{(1)}$
(iii) Lognormal distribution, LN(0,1)

MATLAB R2018a programming language is used for all types of calculations. The number of simulation replications was 10000 for each case. Random samples were generated from each of the above mentioned distributions with Cp =1.0 for size calculation and C_p = 1.33, 1.67 and 2.00 respectively for power calculations. We consider sample size, n=10, 20, 30, 50, 70, 90 and 100 and B=2000, bootstrap samples. The most common significance level (α=0.05) is used for estimating the size and power of the selected tests. Results are tabulated in Tables 3.1- 3.12 for N(50,1) distribution with Cp = 1.0, $\chi^2_{(1)}$ distribution with Cp=1.0, Lognormal(0,1) with Cp=1.0, N(50,1) distribution with Cp = 1.33, $\chi^2_{(1)}$ distribution with Cp=1.33, Lognormal(0,1) with Cp=1.33, N(50,1) distribution with Cp = 1.67, $\chi^2_{(1)}$ distribution with Cp=1.67, Lognormal(0,1) with Cp=1.67, N(50,1) distribution with Cp = 2.00, $\chi^2_{(1)}$ distribution with Cp=2.00, Lognormal(0,1) with Cp=2.00, respectively.

3.2. Results and Discussions

In this section, we will discuss the results of the simulation study, which test statistics have sizes close to the nominal level and also have good powers in finite samples. In Tables 3.1 to Table 3.3 (See Figure 3.1 for better understanding), we have reported simulated sizes when data were generated from the normal $\chi^2_{(1)}$ and LN(0,1) distribution respectively. In the Table 3.1, We have assumed USL = 53 and LSL =47 to calculate the sample Cp. Our simulation results show that the classical test have sizes close to the nominal level as compare to other considered tests. The adjusted classical test sizes found higher than the nominal level for small sample sizes. When n increases, it is noticeable that estimated sizes are going to converge with the 5% nominal level. The large sample test and the augmented large sample tests have better size properties as compared to the adjusted classical test. It means that these two tests sizes are close to the nominal level as comparing to the adjusted classical test. Overall, the robust test has better size properties compared to the adjusted classical test. However, for the 5% and 10% trimmed data we have almost observed consistent patterns of sizes for this test. When n increases, it is observed that sizes are approaches to the nominal level for bootstrap method. Overall, for symmetric distribution, the classical test can be recommended for all sample sizes. The augmented large sample test, robust test and the bootstrap test can be used only for large sample sizes.

Table 3.1. Empirical Sizes for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the N (50,1) Distribution with Skewness 0 and Cp = 1.0

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.0526	0.1358	0.1086	0.1924	0.1386	0.1338	0.0020
20	0.0472	0.1276	0.0930	0.1210	0.1180	0.1194	0.0082
30	0.0518	0.1138	0.0778	0.0924	0.1048	0.1034	0.0210
50	0.0472	0.1070	0.0752	0.0780	0.0976	0.1006	0.0374
70	0.0392	0.0918	0.0730	0.0636	0.0814	0.0810	0.0408
90	0.0506	0.0716	0.0674	0.0684	0.0714	0.0710	0.0421
100	0.0488	0.0622	0.0700	0.0558	0.0656	0.0621	0.0485

Table 3.2. Empirical Sizes for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the $\chi^2_{(1)}$ Distribution with Skewness 2.828 and Cp = 1.0

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.1420	0.0528	0.0328	0.0376	0.0990	0.0942	0.0856
20	0.1238	0.0702	0.0366	0.0403	0.0966	0.0638	0.0806
30	0.1198	0.0642	0.0384	0.0426	0.1040	0.0674	0.0770
50	0.1118	0.0654	0.0386	0.0417	0.1016	0.0630	0.0780
70	0.1058	0.0644	0.0482	0.0433	0.0901	0.0658	0.0602
90	0.0981	0.0646	0.0581	0.0438	0.0830	0.0630	0.0746
100	0.0911	0.0600	0.0662	0.0444	0.0766	0.0738	0.0603

In Table 3.2, we have reported estimated sizes for all considered tests when data are generated from the moderately well-known skewed distribution, namely the Chi-square distribution with 1 df with skewness 2.828. We have considered USL = 5.243 and LSL = -3.243 to estimate the sample Cp. We have observed that the classical test has higher sizes than the other considered tests. All tests have better size properties than the classical test. The adjusted classical test, the large sample test and the augmented test have better size properties than the other considered tests. Among these three tests, overall, the augmented test has sizes close to the nominal level. The bootstrap test has better size properties than the

5% and 10% trimmed robust test. It is evident that the large sample and the augmented large sample tests can be recommended according to the size properties in finite samples.

Table 3.3. Empirical Sizes for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the Lognormal (0, 1) Distribution with Skewness 6.185 and Cp = 1.0

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.1772	0.0422	0.0462	0.0511	0.0454	0.0464	0.0175
20	0.1592	0.0346	0.0418	0.0527	0.0834	0.0868	0.0341
30	0.1360	0.0310	0.0388	0.0529	0.0996	0.0832	0.0494
50	0.1214	0.0228	0.0351	0.0540	0.0900	0.0826	0.0577
70	0.1178	0.0234	0.0332	0.0545	0.0924	0.0916	0.0574
90	0.1104	0.0336	0.0302	0.0540	0.0962	0.0878	0.0522
100	0.1042	0.0346	0.0297	0.0548	0.0940	0.0834	0.0572

We have reported the size properties of the test statistics when data are generated from the highly skewed distribution in Table 3.3. We have considered LSL = -4.835 and USL = 8.132 to compute the sample Cp. Our observation from the simulation studies for the DGP is that the classical test has far size distortion from the nominal sizes for all considered sample sizes. The other test sizes are close to the nominal level, meaning these test sizes have good size properties in finite samples. Overall, our simulation test results show that the augmented large sample test sizes have better size properties as compared to other tests for highly skewed distribution.

Table 3.4. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the N (50,1) Distribution with Skewness 0 and Cp = 1.33

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.3832	0.3622	0.2146	0.3362	0.4906	0.4992	0.0392
20	0.5918	0.5106	0.3320	0.4162	0.8894	0.6168	0.1290
30	0.7418	0.6108	0.4138	0.5588	0.9004	0.7082	0.1636
50	0.8814	0.7370	0.5428	0.5924	0.9844	0.8408	0.1804
70	0.9550	0.8144	0.5940	0.6004	0.9978	0.9058	0.2358
90	0.9808	0.8742	0.7954	0.6800	0.9996	0.9414	0.4852
100	0.9874	0.8836	0.9043	0.9103	1.0000	0.9604	0.4988

In the Table 3.4, we have reported the estimated powers of our considered tests when data are generated from the N (50,1) distribution and Cp = 1.33. Our observation from this table is that all considered tests have reasonable powers except the bootstrap approach. The 5% trimmed robust approach has better power properties, followed by the classical test and the augmented large sample test.

Figure 3.1. Simulated Sizes for all Considered Test Statistics and for all n

Table 3.5. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the $\chi^2_{(1)}$ Distribution with Skewness 2.828 and Cp = 1.33

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.3004	0.3816	0.1858	0.2316	0.2440	0.2358	0.2414
20	0.4298	0.4408	0.3042	0.2830	0.4992	0.4962	0.2454
30	0.4974	0.5846	0.4646	0.3570	0.4232	0.4192	0.3190
50	0.6134	0.6568	0.6306	0.5038	0.5274	0.5200	0.4814
70	0.6962	0.6956	0.7220	0.6838	0.5914	0.6056	0.5600
90	0.7528	0.7682	0.8432	0.7564	0.6490	0.6464	0.5956
100	0.7982	0.8456	0.8510	0.8307	0.7480	0.6124	0.6010

The Table 3.5 reports the estimated powers when data are generated from the Chi-Square distribution with 1 df. We observed that for the moderately skewed distribution, the large sample test is more powerful, followed by the adjusted classical test when sample sizes are large. While, adjusted classical test performed better than rest of the tests for relatively small sample sizes. The classical test and 5% trimmed tests have less power as compared to the N (50,1) distribution. Also, it is observed that the bootstrap approach is more powerful as compared to the symmetric DGP.

Table 3.6. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the Lognormal (0,1) Distribution with Skewness 6.815 and Cp = 1.33

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.2156	0.1010	0.1150	0.1682	0.1202	0.1238	0.3118
20	0.2978	0.1256	0.1596	0.1978	0.2568	0.2622	0.3302
30	0.2960	0.1220	0.1698	0.2364	0.2622	0.2720	0.4358
50	0.4400	0.2014	0.2404	0.3656	0.3778	0.3758	0.5296
70	0.5036	0.2150	0.2930	0.3994	0.3890	0.3810	0.6636
90	0.5460	0.2658	0.3676	0.4104	0.4696	0.4736	0.7046
100	0.5782	0.2846	0.4554	0.5816	0.5310	0.5376	0.7186

In the Table 3.6, we have tabulated simulated powers when data are generated from the highly skewed distribution and the population Cp value of 1.33. We observed from our findings that as comparing to the symmetric distribution and the moderately skewed distribution, all tests are less powerful. Among all, the bootstrap test has better power properties, followed by the classical test, the augmented large sample test, the robust test and so on.

Table 3.7. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the N (50,1) Distribution with Skewness 0 and Cp = 1.67

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.7280	0.5866	0.4209	0.5386	0.7474	0.7622	0.1668
20	0.9276	0.7968	0.5402	0.6014	0.9856	0.9078	0.2202
30	0.9826	0.8942	0.6400	0.7302	0.9950	0.9600	0.4530
50	0.9988	0.9646	0.6907	0.7500	0.9998	0.9954	0.7140
70	1.0000	0.9856	0.7520	0.8400	1.0000	0.9992	0.7996
90	1.0000	0.9948	0.8900	0.9000	1.0000	0.9998	0.8916
100	1.0000	0.9974	0.9230	0.9312	1.0000	1.0000	0.9258

In the Table 3.7, we have stated the estimated powers of tests for the DGP N(50,1) and Cp = 1.67. Our remarks from this table is that all considered tests have good powers as compared to the Table 3.4. Both proposed 5% and 10% robust approaches performed better than the rest of the test statistics for all sample sizes.

Table 3.8. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the $\chi^2_{(1)}$ Distribution with Skewness 2.828 and Cp = 1.67

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.4704	0.2800	0.2328	0.1364	0.3916	0.3990	0.3328
20	0.6568	0.4404	0.2810	0.2766	0.7154	0.7090	0.3934
30	0.7522	0.5732	0.3364	0.3520	0.7002	0.6926	0.4316
50	0.8858	0.6610	0.5340	0.5724	0.8428	0.8580	0.5426
70	0.9010	0.7418	0.6510	0.6889	0.9166	0.9168	0.8126
90	0.9136	0.8108	0.7902	0.8034	0.9550	0.9518	0.6988
100	0.9222	0.8188	0.8304	0.8520	0.9812	0.9486	0.7814

In the Table 3.8, we have reported the estimated powers when data are generated from the moderately skewed distribution and Cp=1.67. We witnessed that both trimmed 5% and 10% robust tests are more powerful than the other tests, followed by the classical, large sample test and augmented large sample test and so on. Also, it is observed that the bootstrap approach is more powerful as compared to the symmetric DGP.

Table 3.9. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the Lognormal (0,1) Distribution with Skewness 6.815 and Cp = 1.67

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.3308	0.2014	0.2936	0.3438	0.2324	0.2240	0.2352
20	0.4698	0.2796	0.3132	0.3998	0.4804	0.4840	0.3266
30	0.5696	0.3694	0.3938	0.4362	0.4212	0.3908	0.4580
50	0.6896	0.4434	0.4626	0.4944	0.4960	0.4974	0.6564
70	0.7764	0.5130	0.5144	0.5630	0.5596	0.5682	0.7146
90	0.8466	0.6098	0.6384	0.6394	0.6254	0.6236	0.8802
100	0.8728	0.6034	0.6564	0.8352	0.7172	0.6918	0.8990

Table 3.9 presents the powers when data are generated from the highly skewed distribution and for Cp value of 1.67. We observed similar patterns of powers, meaning all tests are less powerful and, among all, the bootstrap test has better power properties, followed by the classical test, the augmented large sample test, the robust test.

Table 3.10. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the N (50,1) Distribution with Skewness 0 and Cp = 2.00

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach		Bootstrap Approach
					5%	10%	
10	0.8944	0.7548	0.5208	0.6012	0.8854	0.8864	0.2614
20	0.9896	0.9272	0.6100	0.6570	0.9986	0.9990	0.4358
30	0.9992	0.9736	0.7080	0.7304	0.9998	0.9994	0.5242
50	1.0000	0.9956	0.8020	0.8323	1.0000	1.0000	0.9236
70	1.0000	0.9984	0.8823	0.9033	1.0000	1.0000	0.9728
90	1.0000	0.9994	0.9329	0.9498	1.0000	1.0000	0.9920
100	1.0000	1.0000	0.9610	0.9821	1.0000	1.0000	0.9958

In the Table 3.10, we have described the estimated powers of the tests when data are generated from the symmetric distribution and for the excellent quality control value Cp 2.00. We found that all the considered tests have very good power properties. The bootstrap approach powers significantly increased for this simulation flowchart. Overall, proposed robust approaches and classical test performed equivalently and better than rest of the tests.

Table 3.11. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the $\chi^2_{(1)}$ Distribution with Skewness 2.828 and Cp = 2.00

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach 5%	Robust Approach 10%	Bootstrap Approach
10	0.6206	0.4452	0.3584	0.3778	0.5372	0.5200	0.4184
20	0.8026	0.6506	0.6360	0.6732	0.8492	0.8490	0.4192
30	0.8978	0.7318	0.7552	0.7744	0.8650	0.8578	0.6650
50	0.9506	0.8518	0.8706	0.8730	0.9548	0.9624	0.8116
70	0.9614	0.9014	0.9201	0.9308	0.9874	0.9866	0.8486
90	0.9674	0.9374	0.9560	0.9602	0.9966	0.9944	0.9016
100	0.9680	0.9412	0.9625	0.9709	0.9986	0.9964	0.9462

The Table 3.11 reports the simulated powers of tests when data are generated from the moderately skewed distribution (chi-square distribution with 1 df) and Cp=2.00. We found that all the considered tests have very good power as compared to the Tables 3.5 and 3.8. Overall all, the trimmed 5% robust test performed the best, followed by the trimmed 10% robust test, classical test, the augmented large sample test and so on.

Table 3.12. Empirical Powers for Testing H_0: Cp≤1.0 vs. H_1: Cp>1.0 when Data are Generated from the Lognormal (0.1) Distribution with Skewness 6.815 and Cp = 2.00

Sample size	Classical test	Adjusted classical test	Large sample test	Augmented large sample test	Robust Approach 5%	Robust Approach 10%	Bootstrap Approach
10	0.4290	0.3014	0.3268	0.3644	0.3370	0.3302	0.2592
20	0.6342	0.4590	0.4798	0.4976	0.6396	0.6496	0.4968
30	0.7488	0.5724	0.6268	0.6648	0.6528	0.6748	0.6410
50	0.8684	0.7156	0.7480	0.8282	0.7794	0.7830	0.8436
70	0.9006	0.7930	0.8036	0.8356	0.8572	0.8586	0.8618
90	0.9130	0.8154	0.8308	0.8552	0.9168	0.9026	0.9298
100	0.9254	0.8420	0.8706	0.9046	0.9446	0.9088	0.9449

Finally, to observe the power properties of our selected tests, we have generated random samples from the highly skewed distribution, namely, the lognormal distribution with a Cp value of 2.00. Results are tabulated in the Table 3.12. From Table 3.12 it is evident that the bootstrap approach is more powerful than other considered tests. The 5% trimmed robust approach has better power properties than the classical test at the 5% nominal level.

4. Some Concluding remarks

This paper proposes and considers some test statistics for testing the population process capability ratio and compares their performances under the same simulation conditions but various kinds of distribution such as symmetric and skewed distributions. Our simulation results show that the proposed test statistics have sizes close to the 5% nominal level and also have good powers in finite samples as compared to the existing test statistics. We believe that findings of this paper will contribute to process capability literature, and it will be helpful to choose a test statistic when some researchers are interested in testing the population process capability index.

Dedication. Authors are dedicating this paper to Professor Keshab Chandra Bhuyan, who is currently a Professor at the American International University and former Professor in the Department of Statistics at Jahangirnagar University for his constant inspiration and affection during the author's student life - which motivated them to reach their present positions.

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On Some Examples of Parametric Prediction Interval for Reliability with Exponential and Weibull Distribution

Abdul Hannan Chowdhury*
School of Business and Economics, North South University, Dhaka, Bangladesh

Abstract

Despite their practical importance, prediction intervals have received little attention in texts on statistics, quality control, and reliability analysis *as well as* in life testing experiments, except in relation to regression analysis. In this p*aper*, an attempt has been made to provide a comprehensive presentation of important prediction intervals and to provide numerical examples of their application in the context of reliability. It is well regarded that the Weibull distribution is a life testing distribution and a widely known distribution in reliability and survival analysis. Nevertheless, exponential distribution is a special case of the Weibull, the case which corresponds to constant failure rate. The purpose of this paper is to discuss parametric prediction intervals for reliability when the form of the distribution is exponential or Weibull. The study would cover some of the important prediction intervals with relevant examples.

1. Introduction

In reliability and life testing experiments, prediction intervals, which use the results of a past sample, provide useful information about the realization of a random variable in a future sample from the same distribution. That is, a prediction interval is an interval which uses the results of a past sample to contain the results of a future sample from the same population with a specified probability that serves different purposes. For example, one might wish to predict the number of product failures which will occur in a future period using past data.

Suppose $X_1,X_2,...,X_n$ denote an ordered random sample of size n drawn from a population of size (n+k), say, X_1, $X_2,...,X_n$, X_{n+1}, $X_{n+2},...,X_{n+k}$. Now, if we consider a second (future) sample of size k, X_{n+1}, $X_{n+2},...,X_{n+k}$ from the same population where our interest is to make a probability statement for the future sample based on the information of the past sample. A prediction interval, in contrast to a confidence interval or tolerance interval, could be applicable in such a situation. It is common practice to compute a confidence interval for the population parameter such as for population mean. Sometimes a confidence interval is desired for a future observation itself, rather than its mean. In this case the confidence interval must be somewhat wider to allow for the variation of the variable itself about its mean. Since the interval is for a variable, rather than a parameter, it is sometimes referred to as a prediction interval, instead of a confidence interval. Furthermore, a p-level prediction interval for future observation; may also be

* Corresponding author: hannan.chowdhury@northsouth.edu

interpreted as a p-expectation tolerance interval. A prediction interval can easily be distinguished from a confidence interval for an unknown population parameter (such as the population mean) and a tolerance interval to contain a specified proportion of the population. For further details about confidence interval, tolerance interval and prediction interval consult Hahn [1970, 1972]. The main goal of the study is to demonstrate parametric prediction intervals for reliability when the form of the distribution is either exponential or Weibull. This study would provide a wide-ranging presentation of important prediction intervals with relevant examples in the context of reliability.

2. Review of Literature

One of the earliest papers on prediction intervals is Baker (1935). Since then, a large number of papers on prediction intervals have appeared in the literature. An early review paper on the subject is Hahn & Nelson (1973). A comprehensive review paper by Patel [1989] describes the availability of a large variety of prediction intervals for several life distributions. In the literature, a variety of prediction intervals is available for normal and exponential distributions. For the exponential distribution alone, we figured out prediction intervals by Hahn (1975), Kaminsky (1977), Kaminsky & Nelson (1974), Lawless (1971, 1972, 1977), Hahn & Meeker (1991), Nelson (1970), and others are prominent. For many of these prediction intervals, factors for calculating prediction limits are generally tabulated. However, there are some important prediction intervals for which no such tabled values are available. One of the pioneer papers on prediction intervals for a such case based on one parameter exponential distribution obtained by Lawless (1971). Since then, this model has been investigated extensively, and several prediction intervals covering diverse situations are now available for it.

In different kinds of literature, parametric and non-parametric prediction intervals have been discussed extensively by many authors. Parametric prediction intervals are intervals obtained when the form of the population is known such as normal, exponential and are discussed in papers written by Chew (1969), Hahn (1970), Hahn & Nelson (1973), Hall & Prairie (1973) and Hall, Prairie & Motlagh (1975). On the other hand, non-parametric prediction intervals are intervals obtained when the form of the distribution is unknown and these are discussed in papers written by Danzinger & Davis (1964), Lawless (1971), and Nelson (1963). But, not too many papers have discussed prediction intervals in the context of reliability.

Hsieh (1997) computed quantiles related to prediction intervals for future Weibull order statistics using a conditional method for two scenarios: (i) if only previous independent failure data are available, and (ii) if both previous independent failure data and early-failure data in ongoing experiment are available. Note that quantiles for constructing prediction intervals depend on ancillary statistics of observed data while using the conditional method. Hsieh (1996) utilized the identical method to get prediction intervals for future observations, based on only early-failure data of a current experiment. Hsieh (1997) extended the prediction problem to the case of using both previous independent data and early-failure data of the ongoing experiment. Comparisons have been made for interval widths of different parameter estimators forming prediction intervals in different ways.

Jiang & Zhang (2002) considered prediction intervals for a future observation in the context of mixed linear models assuming that the future observation is independent of the current ones using a distribution-free method. They showed that for standard mixed linear models, a simple method based on the (regression) residuals works well for constructing prediction intervals. Hahn & Meeker (1991) reviewed and compared three types of statistical intervals as the confidence interval, the prediction interval, and the tolerance interval. Note that distribution-free methods play a more significant role in prediction intervals than they do in confidence intervals, especially for large samples. Wu (2015) suggested the general weighted moments' estimators (GWMEs) of the scale parameter of one-parameter exponential distribution based on a multiply type II censored sample to construct the prediction intervals for future observations. Nevertheless, Wu (2016) proposed the prediction interval for future waiting times or inter-arrival time to demonstrate the prediction intervals based on GWMEs. Here, the objective was to investigate the utilization of GWMEs in constructing a pivotal quantity and to find out the prediction interval of future waiting times or inter-arrival times between the two consecutive future observations.

However, one of the major objectives of this study is to provide a comprehensive presentation of important parametric prediction intervals and to provide numerical examples of their application in the context of reliability when the form of the distribution is exponential or Weibull. Since the natural logarithm of a variable with a Weibull distribution has an extreme value distribution. Therefore, prediction intervals for the Weibull distribution found under this study may also be used for the extreme value distribution. This was discussed by Mann & Saunders (1969) and Antle & Rademacher (1972).

3. Prediction Interval

Suppose in a situation in which a sample of size n, $X_1, X_2,...,X_n$ is taken from the population under consideration with unknown parameters and also suppose that a second (future) sample of size k, $X_{n+1}, X_{n+2},...,X_{n+k}$ is taken from the same population. Now let us suppose that $g(X_{n+1}, X_{n+2},...,X_{n+k})$ be some statistic (function of the observations) for the second sample of size k and $g_1(X_1, X_2,...,X_n)$, $g_2(X_1, X_2,...,X_n)$ which are the functions of the first sample of size n. Thus, a two-sided 100γ % prediction interval to contain the future statistic $g(X_{n+1}, X_{n+2},...,X_{n+k})$ with probability γ (specified) is an interval with lower and upper limit $g_1(X_1, X_2,...,X_n)$ and $g_2(X_1, X_2,...,X_n)$ which are the functions of the observations in the first sample, such that the interval encloses the future statistic with probability γ, that is,

$$P[g_1(X_1, X_2,...,X_n) < g(X_{n+1}, X_{n+2},...,X_{n+k}) < g_2(X_1, X_2,...,X_n)] = \gamma \tag{1}$$

for any possible values of the unknown parameters of the underlying distribution. However, the interpretations of such prediction intervals are as follows: Suppose a 100γ% prediction interval for a future sample statistic is calculated from past samples for many such pairs of past and future samples. Then the interval will enclose the future sample statistic in a fraction γ of the pairs of samples in the long run. That is, such intervals enclose the corresponding future statistic with probability γ.

4. Simultaneous Prediction Interval

Simultaneous prediction intervals may be defined for k statistics, g_l, g_2...,g_k, each a separate function of the observations in k independent future samples from the same distribution as a first sample $X_1,...,X_n$. Two-sided simultaneous prediction intervals to contain g_l, g_2....,g_k, with probability γ are intervals with lower and upper endpoints $g_{i1}(X_1,...,X_n)$ and $g_{i2}(X_1,..., X_n)$, i = l,..., k, which are functions of the observations in the first sample, such that the intervals each enclose the corresponding future statistics with probability γ; that is,

$$P[g_{11} \leq g_1 \leq g_{l2}, \text{ and } \cdots \text{ and } g_{kL} \leq g_k \leq g_{k2}] = \gamma \qquad (2)$$

for any possible values of the unknown parameters of the underlying distribution. One-sided simultaneous prediction intervals are similarly defined. However, such simultaneous prediction intervals have the following interpretation: Suppose for many sets of a single past sample and k future samples, 100γ% simultaneous prediction intervals are calculated for sample statistics for each of k future samples. Then all of the k intervals will enclose their respective sample statistics in a fraction γ of the sets of samples in the long run. That is, such intervals enclose all of the corresponding future statistics with probability γ·

5. Problem Statement

Reliability studies and life testing experiments mostly deals problem-related to failure data. Let us suppose that in an experiment some prior failure information is given and one would like to obtain information on the next failure times or to predict the range of the future failure times. Assume that the failure times of a system that occurs from an exponential or from a single Weibull process and the successive failure times x_1, x_2,....,x_n have been recorded. The vital question is that we are concerned about the next failure occur time. In this situation, prediction intervals in contrast to confidence or tolerance intervals are appropriate. That means, a prediction interval for future failure time x_{n+1}, x_{n+2},....,x_{n+k} (in general), would be quite applicable. This implies that we may use the result of a past sample to construct an interval that will contain the results of a future sample from the same population with a specified probability. Throughout it is assumed that both the past and the future samples are obtained with simple random sampling from the same population. The validity of prediction intervals depends strongly on this key assumption.

6. Prediction Interval When Lifetime Follows One Parameter Exponential Distribution

Suppose, in a life testing experiment involving items whose life time's follow an exponential distribution and our problem is to predict the rth ordered observation X_r in a sample of n from the same distribution, based on the observed values of the first k ordered observations from the sample ($k < r \leq n$). Now, suppose that $X_1 \leq X_2 \leq ..... \leq X_n$ are ordered observations in a sample of n from the exponential with mean $1/\theta$, having density,

$$f(x; \theta) = \theta e^{-\theta x}, \qquad \theta > 0, x > 0 \tag{3}$$

Let $S_k = \Sigma X_i + (n-k)X_k$ and the variate $u = u(k,r,n)$ for given $k < r \leq n$ and u is defined by

$$u = (X_r - X_k)/S_k.$$

While deriving density function of u we note two well-known results (see Epstein and Sobel, 1953) concerning ordered observations from an exponential distribution:

(i) the variates $w_1 = nX_1$, $wi = (n-i+1)(X_i - X_{i-l})$, $i = 2,...,n$ are independently distributed with density (3), and (ii) $2\theta S_k = 2\theta \Sigma wi$ is distributed as χ^2 with 2k degrees of freedom.

It then follows rather easily that $\theta(X_r - X_k)$ and θS_k are independently distributed and that $u = (X_r - X_k)/S_k$ has a distribution not involving θ. The probability density function of u is found as

$$f(u) = k/B(r-k,n-r+1) \sum_{i=0}^{r-k-1} (r-k-1)\, (-1)^i [1 + (n-r+i+1)u]^{-k-1}\ ; \quad (u > 0) \tag{4}$$

where , $B(a, b) =(a-1)!\,(b-1)!/(a+b-1)!$ when a, b are positive integers. Integration yields

$$P(u \geq t) = k/B(r-k,n-r+1)[\sum_{i=0}^{r-k-1} (r-k-1)\, (-1)^i/(n-r-+i+1)][1 + (n-r+i+1)t]^{-k} \quad (u > 0)$$

$$i = P(t; k,r,n) \tag{5}$$

when the distribution function of u is given by $F(t) = 1 - P(u \geq t)$.

However, probability statements about u provide prediction statements on X_r, on the basis of observed X_k, S_k,. For example, the statement $P(u \leq t_o) = \alpha$ yields prediction statement,

$$P(X_r, \leq X_k + t_o S_k) = \alpha, \tag{6}$$

giving a (one-sided) 100α% prediction interval for X_r.

However, two additional remarks concerning the evaluation of the above probabilities:

(i) In the important case where r = n (that is, we wish to predict the largest observation on the basis of the k smallest), expression (5) can be expressed as

$$P(t; k,r,n) = 1 - \sum_{i=0}^{n-k} (n-k)\,(-1)^i[1+it]^{-k} \tag{7}$$

hence, the distribution function of $u_1 = u(k, n, n)$ is given by

$$P(u_1 \leq t) = \sum_{i=0}^{n-k} (n-k)\,(-1)^i[1+it]^{-k} \tag{8}$$

(ii) In the special case where k = r-1, Epstein and Sobel's (1953) results, $u_2 = u(r\text{-}1, r, n) = (r-1)(n-r+1)(X_r-X_{r-1})/S_{r-1}$ is an F variate with (2, 2r-2) degrees of freedom, so that appropriate probability statements can be read from standard tables of the F distribution.

Case 1: A Life Test where All Units Are Observed Until Failure:

Consider a life test with n units and whose lifetimes follow the same exponential distribution, are put on test simultaneously, and where all units are observed until failure. We can provide a prediction interval for the largest lifetime X_n on the basis of the k smallest lifetimes $X_1 < X_2 < \cdots < X_k$; X_n is in this case the total elapsed time required to complete the test.

Numerical Example 1

Suppose that 10 items, whose lifetimes are distributed according to the same exponential distribution, are on test simultaneously, and that the first four items to fail to do so at times 30, 90, 120, 170 hours. For n = 10, k = 4 we can find $P(u_1 \leq 2.10)$ is very nearly 95%. Since $X_4 = 170$ and $S_4 = 1430$, this yields the prediction statement $P(X_{10} \leq 170 + (2.10)\ 1430) = P(X_{10} \leq 3173) = .95$. That is, we can be (approximately) 95% confident that the total elapsed test time will not exceed 3173 hours.

Case 2: A Life Test Where Testing Is Terminated After the rth Failure:

Consider a life testing situation similar to that in the above section, but suppose that it had been decided beforehand to terminate the test after the fifth failure.

Numerical Example 2

On the basis of the first four failure times, we can compute, say, an upper 95% prediction limit for X_5. With r = 5, k = 4, n = 10, we consider $u = (X_r - X_k)/S_k$; using equation (5) or noting that in this case 24u is an F variate with (2, 8) degrees of freedom, we find that $P(u \leq .1858) = .95$. Given the observed values $X_4 = 170$, $S_4 = 1430$, this yields the prediction statement

$$P(X_5 \leq 436) = .95.$$

We can be 95% confident that the fifth failure will occur before 436 hours.

7. Prediction Interval Based on Ranges when Lifetimes Follows Two Parameter Exponential Distribution.

Let $R_o(n)$ be the sample range of the lifetimes when n items are put on a life test (without replacement). Similarly, let $R_f(k)$ be the future s-independent sample range of the lifetimes when k items are put on a similar life test. Then the first prediction interval that we have obtained could be used to predict $R_f(k)$ on the basis of the observed $R_o(n)$.

Let, the lifetimes of all items follows a two-parameter exponential distribution, then the two parameter exponential distribution is:

$$f(x;\theta,\beta) = (1/\theta)\exp[(x-\beta)/\theta], \text{ for all } x \geq \beta.$$

If β is known, the distribution becomes, effectively, the one-parameter exponential. Here, θ = scale parameter (unknown), β = location parameter (known or unknown), and $(X_1 < X_2 < \ldots\ldots < X_n)$, $(X_{n+1} < X_{n+2} < \ldots\ldots < X_{n+k})$ = ordered failure times from two s-independent samples. Since, $R_o(n)$, $R_f(k)$ = ranges of past and future samples respectively, where, $R_o(n) = X_n - X_1$, and $R_f(k) = X_{n+k} - X_{n+1}$.

Also $(X^*_1 < X^*_2 < \ldots\ldots < X^*_n)$, $(X^*_{n+1} < X^*_{n+2} < \ldots\ldots < X^*_{n+k})$ = ordered standardized r.v.'s from two s-independent samples from exponential distribution and $R_o^*(n)$, $R_f^*(k)$ = standardized sample ranges past and future samples respectively. Moreover, $1-\alpha$ = prediction probability.

Now, consider the ratio of two independent ranges, $V = R_f(k)/R_o(n) = R_f^*(k)/R_o^*(n)$. The probability distribution of the r.v. Here, V, does not depend on any parameter(s).The Cdf $H_f(Xj)$ of the r.v. $R_f^*(k)$ is known [see David, 1981, p 12]:

$H_f(X_j) = (1-e^{-xj})^{k-1}$, for $X_j \geq 0$. Similarly, cdf

$H_o(X_i) = (1- e^{-xj})^{n-1}$, for $X_i \geq 0$.

Now consider P $(V \leq v) = P[R_f{}^*(k) \leq v\, R_o{}^*(n)]$ (using binomial theorem, see Colangelo and Patel, 1972).

Let $v_\gamma = v(\gamma;m,n)$ be the γ lower quantile for the Cdf(K).

$$\text{Then, } P\{v_{\alpha 1} \leq R_f(k)/R_o(n) \leq v_{1-\alpha 2}\} = l - \alpha \tag{10}$$

with $0 < \alpha_1 < 1$, $0 < \alpha_2 < 1$ and $\alpha_1 + \alpha_2 = \alpha$. The $(1-\alpha)$ two-sided prediction interval of the future sample range $R_f(k)$ on the basis of the past sample range $R_o(n)$ is:

$$[v_{\alpha 1} R_o(n),\ v_{1-\alpha 2} R_o(n)]. \tag{11}$$

Similarly, $(1- \alpha)$ one-sided prediction intervals of the future sample ranges $R_f(k)$ are:

lower prediction limits for $R_f(k)$ is: $[v_\alpha R_o(n), \infty]$,

and upper prediction limits for $R_f(k)$ is: $[O, v_{1-\alpha 2} R_o(n)]$

For computation of prediction factors consult Colangelo and Patel (1972).

Numerical Example 3

In an accelerated life test, consider the following failure times (in weeks) of 10 transistors having a 2-parameter exponential life distribution: 7, 9, 9, 10, 13, 14, 16, 17, 19, 25. One would like to predict the range of the failure times of a future such test of 15 transistors using a 90% prediction interval.

Then, for n = 10, $R_o(10) = 25-7 = 18$, k = 15, and the equal-tail case of $\alpha_1 = \alpha_2 = 0.05$, we find $v_{o.o5} = 0.464797$ and $v_{o.95} = 3.01029$ from table 1 of Colangelo and Patel, (1972). This provides a 90% two-sided prediction interval for $R_f(15)$ as: $[0.464797 \cdot 18, 3.01029 \cdot 18] = [8.37, 54.19]$.

Similarly, 90% one-sided lower and upper prediction intervals for future sample ranges are:

lower prediction intervals: $[0.569004 \cdot 18, 90) = [10.24, \infty)$

and upper prediction intervals is: $[0, 2.43540 \cdot 18] = [0, 43.84]$.

8. Prediction Interval Based on Waiting Time when Lifetime follows Two Parameter Exponential Distribution:

Let W(i) be the waiting time between failures (i-1) and i when n items are put on a life test (without replacement), (i= 1,2,...,n). Now, we want to obtain a prediction interval which can be used to predict the future waiting time W(s) on the basis of the observed (past) waiting time W(r),$(1 \leq r < s \leq n)$.

Let, W(i) = waiting time between failures (i-1) and i: $W(i) = X_i - X_{i-1}$, i=1,2,....n and w*(i) = standardized waiting time between failures (i-1) and i: $w^*(i) = X_i - X_{i-1}$, i=1,2,....n, also 1-α = prediction probability. Let us consider the ratio,

$$F= (n-s+l)W(s)/(n-r+l)W(r) = 2(n-s+l)W^*(s)/2(n-r+l)W^*(r); (1 \le r < s \le n). \tag{12}$$

The (n-s+l)W*(s) is s-independent of (n-r+ I)W*(r), and both have the two-parameter exponential distribution [see David, 1981]. The random variable F has an F-distribution with 2 degrees of freedom in both numerator and denominator. Let $f\gamma = f(\gamma;2,2)$ be its γ lower quantile: $P(F \le f\gamma) = \gamma$. Since

$$P\{f_{\alpha 1} \le (n-s+l)W(s)/(n-r+l)W(r) \le f_{1-\alpha 2}\}= l- \alpha \tag{13}$$

with $\alpha_1+\alpha_2 = \alpha$ defined as in previous section, we have a (l- α) two-sided prediction interval of a future waiting time W(s) on the basis of the past waiting time W(r):

$$[cW(r)f_{\alpha 1,}\ cW(r)\ f_{1-\alpha 2}] \tag{14}$$

where, c =(n-r+l)/(n-s+l). Similarly (l- α) one-sided lower and upper prediction limits W(s) are:

$$[cW(r)f_{\alpha 1}, \infty) \text{ and } [0, cW(r)\ f_{1-\alpha 2}] \tag{15}$$

Since $f\gamma = f(\gamma;2,2)$ can be found from the F-distribution tables. It should be mentioned that necessary prediction factors can be obtained from known tables and to get these prediction factors see Colangelo and Patel, (1972).

Numerical Example 4

In an accelerated life test, let 15 transistors be put on the test (with replacement) and let failure times have a two-parameter Exponential distribution. Let the failure times for transistors #4 and #5, be 10 and 13 weeks, respectively. One would like to find a 90% prediction interval for the waiting time between future failures #9 and #10.

Here n =15, r = 5, W (5) =13-10 =3, s = 10; and $f_{0.90} = f(0.90; 2, 2) = 9.0$. This provides a 90% one-sided upper prediction limit of: (15-5+1 / 15-10+1) 3 · 9 = 49.5.

9. Prediction Intervals when Lifetimes Follows Weibull Distribution

The procedures for obtaining prediction intervals for a future sample from an exponential distribution can be readily extended to obtain prediction intervals for a sample from a Weibull distribution with a known value of the shape parameter. Mann and Saunders (1969) provided one-sided lower prediction limits for the smallest value in a future sample when samples are taken from a Weibull distribution. They used such a limit as the warranty time for the life of a product. Mann (1970) extended tabulations for such a prediction limit based on a linear combination of three selected order statistics from the first sample.

Suppose the lifetimes of all items follow a three parameter Weibull distribution, then

$$f_x(x) = \beta/\theta[(x-n)/\theta]^{\beta-1} \exp[-(x-n)/\theta)^{\beta}]. \tag{16}$$

The parameters β, θ and n are referred to as shape, scale and location parameters respectively. This can also be expressed as $X \rightarrow W(\theta, \beta, n)$. Consider a single Weibull process such as the failure times of a system, and suppose the successive failure times $x_1,...,x_n$ have been recorded. Perhaps the most natural question concerns when the next failure will occur. This suggested that a prediction interval for x_{n+1}, or more generally for x_{n+m} would be quite useful and meaningful in this framework. A prediction interval is a confidence interval for a future observation. Thus a γ level lower prediction limit for x_{n+m} is a statistic T_L (n, m, y) such that $P[T_L(n, m, y) \leq x_{n+m}] = \gamma$.

Consider first the case m = 1. The limit TL should be a function of the sufficient statistics and the probability must be free of parameters.

Theorem: *Suppose X_n ,...,X_{n+1} denote the first n+1 successive times of occurrence of a Weibull process, and suppose the observed values x_1, ... x_n are available. Then, a lower γ level prediction limit for X_{n+1} is $T_L(n, 1, \gamma) = x_n \exp[(\gamma^{-1/(n-1)} -1)/\beta]$* (see Bain and Engelhardt, 1991).

Numerical Example 5

Crow (1974) provided the following simulated data for k = 3 systems with true common β = 0.5 and common θ = 2.778. The data are actually obtained using time truncation at time 200, but for illustrative purposes suppose failure truncation had been employed.

System 1:
4.3, 4.4, 10.2, 23.5, 23.8, 26.4, 74.0, 77.1, 92.1, 197.2

System 2:
0.1, 5.6, 18.6, 19.5, 24.2, 26.7, 45.1, 45.8, 75.7, 79.7, 98.6, 120.1, 161.8, 180.6, 190.8

System 3:
8.4, 32.5, 44.7, 48.4, 50.6, 73.6, 98.7, 112.2, 129.8, 136.0, 195.8

Now, consider the system 1 data. A lower 90% predicted failure time will be

$$T_L(10, 1, 0.90) = 197.2 \ \exp[(0.90^{-1/9} -1) / 0.51] = 201.8$$

10. Prediction Intervals of Waiting Time with GWME of the Scale Parameter of the One-Parameter Exponential Distribution

To predict the waiting time, the pivotal quantity is measured as $V = (Y_{(j)} - Y_{(j-1)})/\hat{\theta}$, $n-s < j \leq n$ based on the GWME $\hat{\theta}$ of the scale parameter of the one-parameter exponential distribution where the lifetimes Y with pdf given by $f(y) = 1/\theta \exp(-y/\theta)$, $y \geq 0$, $\theta > 0$.

Suppose that $Y_{(r+1)} < \ldots. < Y_{(r+k)} < Y_{(r+k+l+1)} < \ldots. < Y_{(n-s)}$ be the available multiply type II censored sample from the above distribution. The GWME to estimate the scale parameter θ is defined as

$$\hat{\theta} = W_{r+1}Y_{(r+1)} + \ldots + W_{r+k}Y_{(r+k)} + W_{r+k+l+1}Y_{(r+k+l+1)} + \ldots + W_{n-s}Y_{(n-s)} = W_{\sim}^{T}Y_{\sim},$$

where $W_{\sim} = [W_{r+1}, \ldots, W_{r+k}, W_{r+k+l+1}, \ldots, W_{n-s}]^T$ and $Y_{\sim} = [Y_{(r+1)}, \ldots., Y_{(r+k)}, Y_{(r+k+l+1)}, \ldots, Y_{(n-s))}]^T$.

The weights $W_{\sim}$ are determined so that the MSE of the proposed GWME is minimized and the GWME with minimum MSE is obtained as $\theta = W_{\sim}^{T}Y_{\sim}$.

Since $Y_{(1)}/\theta$, …., $Y_{(n)}/\theta$ are the n order statistics from a standard exponential distribution and $\theta\hat{}/\theta = W_{\sim}^{T}/Y_{\sim}\theta$ is a linear combination of n order statistics from a standard exponential distribution, the distribution of pivotal quantity $V = (Y_{(j)} - Y_{(j-1)}/\theta)/\theta\hat{}/\theta$ is independent of θ, $n - s < j \leq n$.

Let V (δ; n, j, r, k, l, s) be the δ percentile of the distribution of V satisfying P(V ≤ V (δ; n, j, r, k, l, s)) = δ. Make use of the pivotal quantity, and the prediction interval of waiting time $Y_{(j)} - Y_{(j-1)}$, $n - s < j \leq n$ is proposed in the following theorem.

Theorem: For multiply type II censored sample $Y_{(r+1)} < \ldots. < Y_{(r+k)} < Y_{(r+k+l+1)} < \ldots. < Y_{(n-s)}$, the prediction interval of waiting time $Y_{(j)} - Y_{(j-1)}$, $n - s < j \leq n$ is $(V(\alpha/2; n, j, r, k, l, s)\hat{\theta}, V(1-\alpha/2; n, j, r, k, l, s)\hat{\theta})$ (for further details see Wu 2016).

Numerical Example 6

Suppose that the time to breakdown of an insulating fluid between electrodes is assumed to be exponentially distributed and recorded at 5 different voltages [6]. To illustrate the prediction interval of waiting time assuming 35 kV, the data with n = 12, r = 2, k = 3, l = 1 and s = 5 and the multiply type-II censored failure times (seconds) are: –, –, 41, 87, 93, –, 116, –, –, –, –, –. The weights are 0.23568, 0.12544, 0.19776, and 0.8058.

Here, the estimated scale parameter is $\hat{\theta} = W_{(3)}Y_{(3)} + W_{(4)}Y_{(4)} + W_{(5)}Y_{(5)} + W_{(7)}Y_{(7)}$
$= 41 * 0.23568 + 87 * 0.12544 + 93 * 0.19776 + 116 * 0.8058 = 132.4406.$

We obtained 95% prediction intervals for future waiting times $Y_{(8)} - Y_{(7)}$, $Y_{(9)} - Y_{(8)}$, $Y_{(10)} - Y_{(9)}$, $Y_{(11)} - Y_{(10)}$, and $Y_{(12)} - Y_{(11)}$ are obtained for V (0.025; 12, j, 2, 3, 1, 5), V (0.975; 12, j, 2, 3, 1, 5) corresponds to [0.0058 & 1.1098] with prediction interval is (0.76816, 146.9826); [0.0073 & 1.3875] with prediction interval is (0.9668, 183.7613); [0.0098 & 1.8605] with prediction interval is (1.2979, 246.4057); [0.0146 & 2.7797] with prediction interval is (1.9336, 368.1451); and [0.0292 & 5.5634] with prediction interval is (3.8673, 736.8200), (See for details Wu 2016).

11. Concluding Remarks

For a single future observation, a prediction interval is an interval that will contain a future observation from a population with a specified coverage probability. Many practical problems require that a past sample be used to construct a prediction interval to contain the results of a future sample. In this paper, the author presented most of the available prediction intervals for life testing experiments especially when the lifetimes follow exponential or Weibull distribution, and illustrated their use to provide a guide to those who require such methods in practical applications. Since the natural logarithm of a variable with a Weibull distribution has an extreme value distribution. Therefore, prediction intervals for the Weibull distribution found under this study may also be used for the extreme value distribution.

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Efficiency of Potato Farming in Bangladesh: Cobb-Douglas Stochastic Frontier Approach

Shamima Sultana*
Department of Economics, Jagannath University, Dhaka-1100, Bangladesh

Md. Moyazzem Hossain
Department of Statistics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

M. Ismail Hossain
North South University, Bashundhara, Dhaka-1229, Bangladesh

Abstract

This study examines efficiency of potato farming at farm-level in Bangladesh by estimating technical (TE), allocative (AE) and economic efficiencies (EE) using farm-level cross-sectional survey data from purposively selected six districts: Brahmanbaria, Dinajpur, Joypurhat, Munshigonj, Narsingdi and Rangpur. This study uses Cobb-Douglas Stochastic Frontier Analysis for measuring TE and after that AE and EE are obtained by applying the Kopp and Diewert stochastic cost decomposition method. Stochastic Frontier Analysis is more applicable to agriculture sector since it considers an error term that had two components: one to account for random effects (luck, weather, fires, measurement error in the output variable, etc.) which are not under the owner's control and another to account for technical inefficiency. An evaluation of factors associated with TE, AE and EE from stochastic frontier analysis reveals that except family size other factors are the significant factors affecting TE, AE and EE. Findings reveal that there is a considerable amount of technical, allocative and economic inefficiencies in potato farming and there is potential for boosting output levels through efficiency improvements, hence improving farm revenue and farm household welfare. This study expose that socioeconomic and infrastructure factors jointly determine the variability of potato output. The importance of education and training in improving farm households' ability to receive and understand information about modern technologies is highlighted in this study. Furthermore, land tenure as well as management policies could be designed to reduce land fragmentation in order to better utilization of fertilizer, irrigation, and land preparation using tractors in particular, because fragmentation creates barriers to operating existing technology efficiently and creates difficulties allocating inputs in a cost-effective manner.

Keywords: Potato farming, Stochastic Frontier Analysis, Technical Efficiency, Allocative Efficiency, Economic Efficiency

1. Introduction

The relative efficiency of agricultural farms in developing countries has attracted a lot of attention in the development literature, and economists have been interested in estimating it. Efficiency is a performance measure and success indicator. Studies of efficiency indicate whether it is possible to increase productivity by improving efficiency through utilizing available resources more efficiently without increasing the resource base or developing and adopting new technologies. If agricultural output needs to

* Corresponding author: shamima@eco.jnu.ac.bd

grow at a sufficiently quick rate to fulfill the requirements for food and raw materials emerging from population growth, agricultural farming efficiency is a critical aspect. Estimation of inefficiency can also assist to decide whether to enhance efficiency or to develop new technology to raise overall productivity.

Potatoes are attracting the attention of policymakers and planners in developing countries due to their shown ability to increase farm incomes, rural employment, and food consumption. It is considered as food as well as cash crop. Consumption is particularly important where large segment of the population who lived in rural areas are low-income consumers. Potato production is labor intensive and thus generate employment at farm level (Abedullah et al., 2002)**.** Rice is staple food in Bangladesh but fluctuation in rice production causes significant effects on food security. To keep overall food and nutrition security of the country potato meets both productive and nutritionally balanced requirements (Azimuddin, et.al., 2009) and play vital role in poverty reduction (Zamil, et.al., 2010). The importance of the potato as a winter (October to March) cash crop in Bangladesh is fast increasing for its potential contribution to food security, nutrition, employment and improvement in the socio-economic status of the rural communities (Alam, et.al., 2012).

As producers refocus on home and international markets, the developed world's potato production for the first time surpassed by the developing countries in 2005 (Hossain & Miah, 2009). Using balanced fertilizer, modern seed varieties, integrated pest management and a sound weather during the growing stage helps to get a record crop. As a result, the level of efficiency of potato farming has important consequences for development strategy. The government of Bangladesh has been also trying to diversify food habits, encourages potato consumption to lessen pressure on rice and export of potato to earn foreign currency. The new inputs as well as technology are crucial elements to increase the efficiency of the potato production. If there are substantial opportunities to increase farming efficiency through more efficiently using the resources of farmers with current technology, more emphasis needs to be given on institutional investments in infrastructure, delivery, extension of systems, farm management services and skills of farmers at the farm level.

There is a "Quiet Revolution" of potato production in Bangladesh (Reardon, *et al.*, 2012). It is ranked just after rice and wheat considering the harvested area, production and consumption. Bangladesh ranked 4^{th} position for production area (ha), 3^{rd} position for production volume (MT), 4^{th} position for production yield (tones/ha) and 3^{rd} position for consumption (kg/capita/year) in Asia. It was ranked 9^{th} position in 2005, which became 8^{th} position in 2012 and 7^{th} position in 2014 (FAO, 2014). Bangladesh has relatively favorable soil and climate condition for potato production. The yield potential of potato is about six to seven times more compared to that of rice and wheat from a unit area (Zamil et al., 2010). In terms of both cultivated area and overall production, it ranked first among all vegetables in Bangladesh (Begum, et al., 2010). It is also the 2nd best food energy source (FAO, 2014). Local and high yielding varieties of potato are being cultivated in Bangladesh. Farmers were interested in growing potatoes since they yielded more and made more money than other crops, and they chose Diamant as the most profitable variety (Hossain et al., 2014). However, to attain the target output level, efficiency necessitates sensible input allocation, which is critical for producers attempting to optimize their production decisions. It also improves the farms' ability to adapt to changing market conditions, rising input costs, economic downturns, and quick technology advancement. Moreover, it is useful for policymakers who want to improve farms' economic performance and competitiveness, as well as promote economic development and sustainable economic practices (Guesmi, 2014).

Several previous studies applied Cobb-Douglas Stochastic Frontier Production Model to various agricultural crops (Baten, et al., 2010; Ferdushi, et al., 2013; Ndubueze-Ogaraku & Ekine, 2015; Hossain, et al., 2015; Hossain, 2016; Sujan, et al., 2017; Shavgulidze, et al., 2017; Dube, et al., 2018; Njiku & Nyamsogoro, 2018; Radhakrishnan & Das, 2019; Mwanguhya & Ekere, 2021). However, a few studies on potato production efficiency (Huda, et al., 2019; Tiruneh, et al., 2017; Sapkota & Bajracharya, 2018; Hamjah, 2014; Shahriar, et al., 2013; Begum, et al., 2010; Hossain, et al., 2008) have been undertaken. Majority of the study have focused on estimating technical efficiency using Stochastic Frontier Analysis. Maganga (2012) shows empirically the technical efficiency of potato producer in Irish. Nyagaka, et al. (2010) assesses the technical efficiency in resource use and identifies the underlying determinants of variation in production efficiency among smallholder potato producers in Kenya.

There is a little information on efficiency, especially technical, allocative and economic efficiency of potato sector in Bangladesh. Therefore, it is necessary to undertake research efforts to estimate the efficiency of potato farming that will ensure better ways and enable to improve potato productivity in a sustainable manner. Efficiency improvement will be an important factor in order to get financial success for farmers and profit gain. Resources have to be optimally utilized in order to get maximum production and income. Estimation of the extent of inefficiency can help in deciding how to raise farming efficiency. Identifying sources of inefficiency plays an important role in designing policies to improve the performance of potato farming. However, efficiency of farmers depends on their experience, level of education, land size and fragmentation, use of modern technology, use of seeds, fertilizer and other inputs. Thus, this study attempts to estimate the farm-level TE, AE and EE of potato sector and to identify sources of inefficiency where improvements can be made and tries to find out how these factors affect the efficiency level of the potato farming at the study area. The rest of the paper outlines as Section 2, describes methodology, Section 3 examines the findings of the study and finally, Section 4 makes the conclusion.

2. Methods and Materials

2.1 Data Collection

For the study area, we select six potato growing districts purposively considering the volume of production. An empirical study of farming efficiency requires availability of farm level data, however, due to the lack of data in the selected study areas, a survey was conducted to gather the required data. A pilot survey was carried out prior to design the main survey. The survey documented two types of information, farm characteristics and farm production data. The target population of the study are the potato growers of the selected study areas. The selected districts are Munshigonj, Narsingdi, Brahmanbaria, Rangpur, Dinajpur and Joypurhat. One upazila from each of the districts and two villages from each upazila were selected. However, Munshigonj is in the top position of the potato production and Joypurhat also produces considerable amount of potato that's why two upazilas were selected from these two districts that is 8 villages were included from 6 districts. In each stage of selection, we chose purposive and convenience sampling because of unavailability of the list of farmers and some farmers were not willing to provide the required information. Since potato farming is more or less homogeneous sample size was disproportionately determined as 25 potato growers of each village totaling 400 potato growers. Data were collected using a structured questionnaire which designed in line with the objectives of the study. Output, land, and labor are measured initially by their original unit (output in kg., land in decimal, and labor in hour) and before going to analysis these variables are converted into BDT (Bangladeshi currency).

2.2 Study Area

The selected districts i.e., Munshigonj, Narsingdi, Brahmanbaria, Rangpur, Dinajpur and Joypurhat are shown in the following Bangladesh geographical map (Figure 1).

2.3 Methods

The technical efficiency is estimated using self-dual Cobb-Douglas Stochastic Frontier Model where the maximum likelihood method is used as a default estimation process. First technical efficiency is calculated in a single stage method in which the technical inefficiency effects are modeled as a function of socio-economic characteristics and infrastructure factors. After that allocative and economic efficiency measures are obtained by applying the Kopp and Diewert (1982) stochastic cost decomposition method. We estimate the Cobb-Douglas stochastic frontier model along with a technical inefficiency effects model considering the truncated and half-normal distribution. This study applies a Cobb-Douglas stochastic frontier Analysis Method. It is self-dual and its dual cost frontier model forms the basis for computing technical, allocative and economic efficiencies. The Cobb-Douglas stochastic frontier specified for this study is as follows:

$$\ln y_i = \beta_0 + \sum_{k=1}^{8} \beta_{ik} \ln x_{ik} + v_i - u_i \qquad (i = 1, 2, \dots, 400) \tag{1}$$

Figure 1. Study area

Now, subtracting v_i from both sides of equation (1) yields

$$\ln \tilde{y}_i = \ln y_i - v_i = \beta_0 + \sum_{k=1}^{8} \beta_{ik} \ln x_{ik} - u_i \tag{2}$$

where, ln: the natural logarithm; y: current year total potato production in ('000) Tk.; x_1: land in ('000) Tk.; x_2: labor in ('000) Tk.; x_3: tilling in ('000) Tk.; x_4: seed in ('000) kg.; x_5: fertilizer in ('000) Tk.; x_6: irrigation in ('000) Tk.; x_7: pesticide in ('000) Tk.; x_8: vitamin in ('000) Tk.; β_0: technical efficiency level; $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8$: coefficients of inputs with respect to output level; v_i: the random component accounts for random variations in output because of factors not under the control of the farmers; u_i: the non-negative random error measures the technical inefficiency relative to the stochastic frontier; $u_i = 0$ indicates the farm lies on the Stochastic Production Function; $u_i > 0$ implies that the farm is inefficient; v_i and u_i are assumed to be independent of each other and also independent of the input vector x and $\tilde{y}_i$: the farms observed output adjusted for the stochastic random noise captured by v_i.

The variance parameters of the model are expressed as:

$$\sigma^2 = \sigma_v^2 + \sigma_u^2 \, ; \gamma = \sigma_u^2 / \sigma^2 \, ; \text{and } 0 \leq \gamma \leq 1. \tag{3}$$

Here, γ = 0 indicates absence of stochastic technical inefficiency turning the stochastic frontier model to the average frontier model and γ = 1 represents absence of the stochastic random error term making the stochastic frontier model a full frontier model and it is considered by Aigner and Chu (1968). The parameter $\lambda = \sigma_u/\sigma_v$ is considered as a measure of the relative variability of two sources of inefficiency. when $\lambda^2 \to 0$, it implies that $\sigma_v^2 \to +\infty$ and /or $\sigma_u^2 \to 0$, which means that the random shocks dominate in the explanation of the inefficiency. When $\sigma_v^2 \to 0$ then the gaps to the frontier are essentially due to the technical inefficiency.

The farm specific technical efficiency of i-th farm is defined as the ratio of the observed output (y_i) to the corresponding frontier output (y*), given the levels of the inputs as:

$$TE_i = \frac{y_i}{y_i^*} = \frac{f(x_{i,}\beta)e^{v_i - u_i}}{f(x_{i,}\beta)e^{v_i}} = e^{-u_i}, (0 \leq TE_i \leq 1) \tag{4}$$

Here, u_i are non-negative truncation of the $N\left(\mu, \sigma_u^2\right)$ distribution and $\mu = z_i\delta_i$, where, z_i is a (k×1) vector of variables which may influence efficiency and $\delta_i$is an (1×k) vector of parameters to be estimated.

The equation (2) constitutes the basis for obtaining the technically efficient input vector x_{ik}^T. The dual stochastic frontier cost function model is analytically derived from the stochastic production model. Algebraically deriving the dual stochastic frontier cost function is the basis for calculating the economically efficient input vector x_{ik}^E. The dual stochastic frontier cost function model is

$$C(p_{ik}, \tilde{y}_i) = \alpha_0 \prod_{k=1}^{8} p_{ik}^{\beta_{ik}\alpha_{ik}} \tilde{y}_i^{\alpha_{ik}} , \tag{5}$$

where, $C(p_{ik}, \tilde{y}_i)$ is cost function, $\alpha_0 = \left(\frac{1}{\beta_0^{\alpha_{ik}}}\right)\left(\sum_{k=1}^{8} \beta_{ik} \,/\, \prod_{k=1}^{8} \beta_{ik}^{\beta_{ik}\alpha_{ik}}\right)$ and $\alpha_{ik} = \frac{1}{\sum_{k=1}^{8} \beta_{ik}}$.

Also, $x_{ik}^E = \frac{\partial C(p_{ik}, y)}{\partial p_{ik}} = x_{ik}^E\ (p_{ik}, \tilde{y}_i) = \alpha_0(\beta_{ik}\alpha_{ik}) \prod_{k=1}^{8} \frac{1}{p_{ik}} p_{ik}^{\beta_{ik}\alpha_{ik}} \tilde{y}_i^{\alpha_{ik}}$. (6)

From the result of stochastic frontier production function (1) we can get technically efficient input vector x_{ik}^T. Multiplying the observed input vector x_{ik}, technically efficient input vector x_{ik}^T and economically efficient input vector x_{ik}^E by the input price vector provides the observed, technically efficient and economically efficient costs of production of the i-th farm equal to $p_{ik}x_{ik}$, $p_{ik}x_{ik}^T$ and $p_{ik}x_{ik}^E$ respectively which compute the TE, AE and EE indices for the ith farm as:

$$\text{TE} = p_{ik}x_{ik}^T/p_{ik}x_{ik},\ \text{AE} = p_{ik}x_{ik}^E/p_{ik}x_{ik}^T \text{ and EE} = p_{ik}x_{ik}^E/p_{ik}x_{ik} . \tag{7}$$

The linear form of the dual cost function can be written as (Ouattara, 2010),

$$\ln\left(C\left(p_{ik},\tilde{y}_i\right)\right)=\alpha_0+\phi_1\ln\left(p_{i1}\right)+\phi_2\ln\left(p_{i2}\right)+\ldots+\phi_8\ln\left(p_{i8}\right)+\varphi\ln\left(\tilde{y}_i\right) \tag{8}$$

where, p_1: price of land; p_2: price of labor; p_3: price of tilling; p_4: price of seed; p_5: price of fertilizer; p_6: price of irrigation; p_7: price of pesticide; p_8: price of vitamin and $\tilde{y}_i$: the farms observed output.

The inefficiency effects model is presented by the following equation

$$U_i = \delta_0 + \delta_1 Z_{i1} + \delta_2 Z_{i2} + \delta_3 Z_{i3} + \delta_4 Z_{i4} + \delta_5 Z_{i5} + \delta_6 Z_{i6} + \delta_7 Z_{i7} + \delta_8 Z_{i8} + \delta_9 Z_{i9} + w_i \tag{9}$$

where, U_i : inefficiency ; Z_1: age of farmers in years; Z_2: education of farmers in years; Z_3: training (dummy); Z_4: experience of potato cultivation in years; Z_5: number of plot; Z_6: access to credit (dummy); Z_7: cold storage facilities (dummy); Z_8: household size (number of family member); Z_9: deweeding or weed uprooting cost in Tk.

3. Results and Discussion

Empirical results of stochastic frontier analysis model i.e. the maximum likelihood estimates of the parameters along with other results are presented in Table 1.

Table 1. Results of the Cobb-Douglas Frontier Model of the Selected Districts

Variables	Parameters	Coefficients					
		BB	N	M	J	D	R
Constant	β_0	3.353 (42.397)	9.860 (4.152)	5.262 (5.235)	10.176 (2.475)	12.894 (3.857)	6.445 (6.422)
Ln of Land	β_1	0.760 (15.684)	0.678 (3.818)	0.533 (2.782)	0.512 (5.571)	0.758 (4.269)	0.662 (3.650)
Ln of Labor cost	β_2	0.515 (3.543)	0.591 (6.679)	0.648 (3.379)	0.448 (2.645)	0.445 (2.982)	0.429 (11.372)
Ln of Tilling cost	β_3	0.187 (2.585)	0.380 (3.036)	0.449 (2.614)	0.299 (1.208)	0.267 (3.862)	0.365 (2.451)
Ln of Seed	β_4	0.201 (7.131)	0.591 (5.123)	0.342 (2.663)	0.482 (2.051)	0.381 (8.692)	0.439 (2.893)
Ln of Fertilizer cost	β_5	0.608 (5.155)	0.163 (5.910)	0.457 (5.902)	0.315 (3.664)	0.356 (2.245)	0.330 (2.917)
Ln of Irrigation cost	β_6	0.361 (4.526)	-	0.240 (2.130)	0.108 (8.033)	0.067 (4.003)	0.263 (3.338)
Ln of Pesticide cost	β_7	0.081 (2.566)	0.149 (2.158)	0.155 (2.613)	0.007 (3.228)	0.074 (2.253)	0.215 (4.508)
Ln of Vitamin cost	β_8	0.056 (2.563)	0.029 (2.506)	0.183 (2.311)	0.009 (3.032)	0.052 (2.621)	0.193 (3.587)
Inefficiency Variables							
Constant	δ_0	1.099 (3.098)	1.109 (2.105)	2.315 (7.196)	2.420 (0.246)	18.263 (3.308)	8.013 (5.013)
Age	δ_1	-0.798 (-3.778)	-0.378 (-4.796)	-0.274 (-8.003)	-0.311 (-2.696)	-0.416 (-2.069)	-0.229 (-7.822)

Variables	Parameters	Coefficients					
		BB	N	M	J	D	R
Education	δ_2	-0.536 (-3.582)	-0.438 (-3.227)	-0.127 (-6.625)	-0.863 (-2.559)	-0.427 (-5.713)	-0.214 (-17.239)
Experience	δ_3	-0.548 (-4.235)	-0.558 (-3.588)	-0.421 (-4.193)	-0.153 (-2.953)	-0.365 (-4.509)	-0.261 (-14.426)
Land Frag-mentation	δ_4	0.231 (15.048)	0.166 (4.445)	0.387 (4.065)	0.297 (6.118)	0.584 (2.866)	0.140 (2.846)
Family size	δ_5	0.056 (0.056)	0.102 (3.894)	0.139 (0.108)	0.125 (0.808)	0.183 (0.837)	0.054 (1.057)
Deweeding	δ_6	-0.103 (-2.914)	-0.606 (-5.037)	-0.222 (-4.744)	-0.415 (-3.850)	-0.049 (-2.227)	-0.820 (-6.758)
Access to Credit (dummy)	δ_7	-	-	-0.121 (-3.929)	-0.534 (-4.451)	-	-0.106 (-2.436)
Cold Storage (dummy)	δ_8	-	-	-0.010 (-3.004)	-0.357 (-2.565)	-	-0.276 (-.2604)
Training (dummy)	δ_9	-	-	-0.104 (-2.758)	-0.741 (-2.244)	-0.670 (-3.775)	-0.217 (-15.313)
Variance Parameters							
$\sigma^2 = \sigma_v^2 + \sigma_u^2$		0.042 (2.729)	0.636 (7.472)	0.345 (3.074)	0.282 (2.589)	0.280 (4.325)	0.081 (6.405)
$\gamma = \left(\sigma_u^2 / \sigma_v^2 \right)$		0.523 (5.463)	0.518 (4.264)	0.569 (5.601)	0.473 (4.960)	0.685 (15.694)	0.675 (9.317)
σ_v^2		0.011	0.221	0.123	0.075	0.118	0.025
σ_u^2		0.031	0.415	0.222	0.207	0.162	0.056
Log-likelihood		460.92	334.45	786.29	555.40	437.17	482.65

Note: values in the parenthesis indicate the value of t-statistic, BB: Brahmanbaria; N: Narsingdi; M: Munshigonj; J: Joypurhat; D: Dinajpur; R: Rangpur.

The results of t-statistic reveal that the estimated coefficients of eight positive parameters are statistically significant at 5% significance level for all study areas. In Munshigonj, the elasticity of labor is comparatively higher than other districts. This is quite obvious for Munshigonj district as most of the young people are living abroad which can cause labor scarcity. A lot of seasonal workers migrate to Munshigonj from rest of the country. We found irrigation; pesticide and vitamin have a relatively small effect. According to the calculated δ-coefficients of the explanatory variables, the farm specific variables included in the inefficiency model impact significantly to the explanation of the technical inefficiency effects in potato farming as a group. The signs of estimated coefficients indicate that these variables cause variation in farm technical efficiency, which has an impact on farms' ability to use existing infrastructure and technology effectively. All coefficients are statistically significant which indicating that there are inefficiency effects in the potato farming in the sample farms and also prevails the random component of the inefficiency effects which makes significant contribution in the analysis of efficiency in potato farming. That is the technical inefficiency effects are a significant component in determining the level and variability of output of potato farming in each district.

The coefficient of education is negative suggesting that the farmers with higher years of schooling having higher technical efficiency (Begum, et.al 2012, Maganga, 2012). The higher educational qualification of a farmer tends to adopt new technology. Farmers with basic literacy may utilize contemporary fertilizers and insecticides, as well as select input combinations. With education and extension services, farmers can be introduced to new technology and techniques. The efficiency of allocative allocation is connected to the level of improved education (Ram, 1980). Similarly, coefficients of age and experience have negative impacts on the inefficiency of the yield of potato production which implies that the older and experienced farmers are more technically efficient. As they grow older, they are more technically sound because of experience (Haider, et.al. 2011; Belete, 2020). If the farmers have cold storage facilities, they do not need to sell their product soon after they harvest. In our study areas, only three districts have cold storage facilities. Study shows that storage facilities negatively affects the efficiency level of the potato production, other three districts such as Brahmanbaria, Narsingdi and Dinajpur have not any cold storage facilities, they need to sell their product soon after they harvest (Hoque, 2011; Mukul, et.al. 2013). During harvest time products are usually fetches lower price. If they can keep the potato at the cold storage, then they will get a good margin.

The credit facilities and training also have similar pattern of impacts on inefficiency level of the potato production because the access to credit and training will be substantially reduce the inefficiency of production and this findings is consistent with previous research (Hossain, et.al., 2008; Haider, et.al., 2011, Abate, et al., 2019; Belete, 2020), the production would be more efficient if the farmer do de-weeding their land when they grow potato because weed hampers the potato production substantially (Khan, et.al., 2008; Shahriar, et.al. 2013). As far as land fragmentation is concerned, technical inefficiency effects are greater for farmers with smaller plot size and this findings is supported by other studies (Balogun, & Akinyemi, 2017; Danquah, et al., 2019; Abate, et al., 2019; Belete, 2020). The higher the smaller plots to be managed it would be difficult to manage the land and especially irrigation and tillage would be difficult with tractor and water pump with electricity. In Narsingdi, Rangpur, Dinajpur and Brahmanbaria districts the lands are smaller and the farmers do have smaller pieces of land and they don't use very much technologically advanced method of cultivation. Since, the coefficient of the family size is not statistically significant, family size has not any significant impact on inefficiency performance of the potato production which is supported by other study (Pandit, et al., 2009).

The returns to scale $\left(\sum_{i=1}^{8} \beta_i\right)$ for Brahmanbaria, Narsingdi, Munshigonj, Joypurhat, Dinajpur and Rangpur are 2.77, 2.58, 3.00, 2.18, 2.40 and 2.90 respectively. This signifies that there is increasing returns to scale in potato production for each district. The estimated value of γ parameter for Brahmanbaria, Narsingdi, Munshigonj, Joypurhat, Dinajpur and Rangpur are 0.52, 0.52, 0.57, 0.47, 0.69

and 0.68 respectively. The variance parameters of the selected districts are 0.04, 0.63, 0.35, 0.28, 0.28 and 0.08 respectively.

In Brahmanbaria we can see that most of the farms (80%) are technologically efficient and falls 1-70% efficiency class while 90% farms fall 60-100% efficiency class as far as AE is concerned but 80% farms falls 1-60% efficiency class in case of EE. In Narsingdi, we can see that most of the farms (95%) are technologically, allocatively and economically efficient and fall 60-100% efficiency class. We can see the same picture in case of Joypurhat, Dinajpur and Rangpur in case of Munshigonj we can see that 80% farms falls 1-60% efficiency class as far as EE is concerned. The results demonstrate that there are considerable variations of efficiency among farms. Summary statistics shows that estimated mean TE, AE and EE are better if we compare with Munshigonj and Brahmanbaria where all three efficiencies i.e., TE, AE and EE are lower which implies that potato farming is technically, allocatively and economically more inefficient than any other districts though Munshigonj is famous for potato production in entire Bangladesh. The results also indicate that there is considerable inefficiency in potato farming and rooms for production gain through efficiency improvement in all the districts especially in Munshigonj and Brahmanbaria [Appendix 1]. The dual cost frontier analytically resulting from the stochastic production frontier for the inefficiency components. The stochastic production function and other related functions are represented in Appendix 2.

Moreover, the factors affecting technical, allocative and economic efficiencies of stochastic frontier analysis model are presented in Appendix 3. The coefficient of farming experience is negative implying that households who are into farming for more number of years tended to more technically efficient which indicates that the majority of farmers improve production performance from skills gained through years of experience in potato cultivation. The coefficient for land fragmentation variable is positive in all the districts and groups as well implying that inefficiency tended to increase with the increase in land fragmentation. Less land fragmented farms are more technically efficient compared to more land fragmented farms because farmers can easily apply modern technologies and which is more economic. The coefficient of family size is also positive in all the districts and groups but not significant implies large or small family size has not significantly effect on farming efficiency. The coefficient of de-weeding, access to credit and cold storage facility also have statistically significant effects on farming efficiency and in some districts it is a very important factor of production and inefficiency depends on this factor. The coefficient of Training through contact with extension workers is estimated negative and an important factor for production and efficiencies for Munshigonj, Rangpur, Dinajpur, Joypurhat group which indicates that farmers who have more contact with extension workers tend to be more technically efficient and operate their farms more efficiently and produce closer to the frontier output. The negative sign of the education variable implies farmers with more schooling tended to be technically more efficient. The education coefficient is considerable. It means that farmers with a greater level of education are less inefficient (or more efficient) than farmers with a lower level of education or none at all [Appendix 3].

4. Conclusion

The empirical results of Cobb-Douglas stochastic frontier show that the sign of the estimated coefficients are all positive which implies that all inputs are important in determining production of potato. The returns to scale for Brahmanbaria, Narsingdi, Munshigonj, Joypurhat, Dinajpur and Rangpur are 2.77, 2.58, 3.00, 2.18, 2.40 and 2.90 respectively indicating that the selected farm households have increasing returns to scale in potato production. The estimates of the variance parameter σ^2 and the parameter γ are statistically significant indicating that there are inefficiency effects in the potato farming of sample farms and the random component of the inefficiency effects make significant contribution in the analysis of efficiency of potato farming. Technical inefficiency effect is a significant component in determining the level and variability of output of potato farming in Bangladesh. The results of the technical inefficiency effects model demonstrate that the combined error term is dominated by technical inefficiency effects. The estimated δ-coefficients of the explanatory variables such as age, education, experience, plot size, etc. imply that the farm-specific factors in the inefficiency model contribute significantly to the explanation of technical inefficiency effects in potato farming as a whole.

This study emphasizes the need of education and training to enhance the ability of farm households to receive as well as understand information regarding advanced technology. Formal education particularly agriculture related education can help the farmers to increase their knowledge about cultivation and cost minimizing input use which can improve allocative efficiency. Extension programs could be utilized to reorient the usage of methods, as well as the time and amount of inputs and production processes. Soil tenure and management regulations could be developed to lessen fragmentation in order to effectively utilize irrigation, fertilizer, and land preparation methods employing tractors in particular. It poses issues in distributing inputs in a cost-effective manner because it creates barriers to employing existing technology efficiently. Credit facility is directly related to efficiency and it need to improve and make easy access to farmers.

This study has taken a sample size of 400 farms. It would be preferable if the data and information collected were based on a larger sample size. Other samples may have different findings. To increase efficiency in potato production, more study and inquiry is required. Many other factors, like management style, experience, input quality, and so on, may be included in further study to identify the efficiency of a farm's.

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Appendix 1. Frequency distribution (%) of farms specific efficiencies for the selected districts

Efficiency Index (%)	Brahmanbaria			Narsingdi			Munshigonj			Joypurhat			Dinajpur			Rangpur		
	TE	AE	**EE**	**TE**	AE	**EE**	**TE**	AE	**EE**	**TE**	AE	**EE**	**TE**	AE	**EE**	**TE**	AE	**EE**
1-40	15	1	34	0	1	2	18	15	55	1	2	2	0	0	0	1	3	7
40-50	18	1	22	0	1	1	10	19	18	2	3	5	0	0	0	1	3	9
50-60	22	3	20	0	3	7	7	5	10	3	8	7	0	2	5	2	5	12
60-70	20	9	2	0	20	30	15	20	5	10	12	10	2	9	27	5	18	30
70-80	5	22	4	5	45	35	12	12	7	12	25	32	6	55	45	17	38	15
80-90	2	40	6	10	20	13	18	14	3	22	30	24	24	20	12	15	17	8
90-100	18	24	12	85	10	12	20	15	2	50	20	20	68	14	11	59	16	19
Total Farms	50	50	50	50	50	50	100	100	100	100	100	100	50	50	50	50	50	50
Summary Statistics																		
Mean	65.76	83.25	50.32	97.82	78.25	75.67	68.75	60.32	38.91	99.07	80.29	79.75	96.78	82.52	79.12	90.95	78.32	70.82
Minimum	25.15	55.5	28.2	90.5	32.6	30.3	21.5	20.3	12.7	90.3	9.5	9.5	78.3	53.1	53.5	45.7	5.3	7.5
Maximum	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
Standard Deviation	20.28	12.32	17.18	5.7	10.25	11.15	25.23	18.15	20.12	1.55	15.35	15.27	5.45	7.47	10.38	12.21	15.17	17.75

Appendix 2. Estimated stochastic production functions

Brahmanbaria

I. Stochastic Production Function:

$\text{Ln } y_i = 3.353 + 0.760 \ln x_{i1} + 0.515 \ln x_{i2} + 0.187 \ln x_{i3} + 0.201 \ln x_{i4} + 0.608 \ln x_{i5} + 0.361 \ln x_{i6} + 0.081 \ln x_{i7} + 0.056 \ln x_{i8}$

or, $y_i = 3.353\, x_{i1}^{0.760}\, x_{i2}^{0.515}\, x_{i3}^{0.187} x_{i4}^{0.201}\, x_{i5}^{0.608}\, x_{i6}^{0.361} x_{i7}^{0.081}\, x_{i8}^{0.056}$

II. Dual stochastic frontier cost function:

$C\,(P_{ik}, \tilde{y}_i) = 3.990 P_{i1}^{0.274}\, P_{i2}^{0.186}\, P_{i3}^{0.068}\, P_{i4}^{0.073}\, P_{i5}^{0.220}\, P_{i6}^{0.130}\, P_{i7}^{0.029}\, P_{i8}^{0.020}\, \tilde{y}_i^{0.361}$

III. Input demand function:

$x_{i1} = \frac{\partial C}{\partial P_{i1}} = 3.990(0.274)\, P_{i1}^{(0.274\text{-}1)}\, P_{i2}^{0.186}\, P_{i3}^{0.068}\, P_{i4}^{0.073}\, P_{i5}^{0.220}\, P_{i6}^{0.130}\, P_{i7}^{0.029}\, P_{i8}^{0.020}\, \tilde{y}_i^{0.361}$

or, $x_{i1} = \frac{1.093\, P_{i2}^{0.186}\, P_{i3}^{0.068}\, P_{i4}^{0.073}\, P_{i5}^{0.220}\, P_{i6}^{0.130}\, P_{i7}^{0.029}\, P_{i8}^{0.020}\, \tilde{y}^{0.361}}{P_{i1}^{0.726}}$

where, i = 1, 2, 3, …, 50, number of farms

Narsingdi

I. Stochastic Production Function:

$\text{Ln } y_i = 9.860 + 0.678 \ln x_{i1} + 0.591 \ln x_{i2} + 0.380 \ln x_{i3} + 0.591 \ln x_{i4} + 0.163 \ln x_{i5} + 0.149 \ln x_{i6} + 0.029 \ln x_{i7}$

or, $y_i = 9.860\, x_{i1}^{0.678}\, x_{i2}^{0.591}\, x_{i3}^{0.380} x_{i4}^{0.591}\, x_{i5}^{0.163}\, x_{i6}^{0.149} x_{i7}^{0.029}$

where, i = 1, 2, 3, …, 50, number of farms.

II. Dual stochastic frontier cost function:

$C(P_{ik}, \tilde{y}_i) = 2.251 P_{i1}^{0.263}\, P_{i2}^{0.229}\, P_{i3}^{0.147}\, P_{i4}^{0.229}\, P_{i5}^{0.063}\, P_{i6}^{0.058}\, P_{i7}^{0.011}\, \tilde{y}_i^{0.387}$

where, i = 1, 2, 3, …, 50, number of farms.

III. Input demand function:

$x_{i1} = \frac{\partial C}{\partial P_{i1}} = 2.251(0.263) P_{i1}^{(0.263\text{-}1)}\, P_{i2}^{0.229}\, P_{i3}^{0.147}\, P_{i4}^{0.229}\, P_{i5}^{0.063}\, P_{i6}^{0.058}\, P_{i7}^{0.011}\, \tilde{y}_i^{0.387}$

or, $x_{i1} = \frac{0.592\, P_{i2}^{0.229}\, P_{i3}^{0.147}\, P_{i4}^{0.229}\, P_{i5}^{0.063}\, P_{i6}^{0.058}\, P_{i7}^{0.011}\, \tilde{y}^{0.387}}{P_{i1}^{0.737}}$

where, i = 1, 2, 3, …, 50, number of farms.

Munshigonj

I. Stochastic Production Function:

$\text{Ln } y_i = 5.262 + 0.533 \ln x_{i1} + 0.648 \ln x_{i2} + 0.449 \ln x_{i3} + 0.342 \ln x_{i4} + 0.457 \ln x_{i5} + 0.240 \ln x_{i6} + 0.155 \ln x_{i7} + 0.183 \ln x_{i8}$

or, $y_i = 5.262 x_{i1}^{0.533} x_{i2}^{0.648} x_{i3}^{0.449} x_{i4}^{0.342} x_{i5}^{0.457} x_{i6}^{0.240} x_{i7}^{0.155} x_{i8}^{0.183}$

where, i = 1, 2, 3, …, 100, number of farms.

II. Dual stochastic frontier cost function:

$C(P_{ik}, \tilde{y}_i) = 4.169\, P_{i1}^{0.177} P_{i2}^{0.215} P_{i3}^{0.149} P_{i4}^{0.114} P_{i5}^{0.152} P_{i6}^{0.080} P_{i7}^{0.052} P_{i8}^{0.061} \tilde{y}_i^{0.333}$

where, i = 1, 2, 3, …, 100, number of farms.

III. Input demand function:

$x_{i1} = \frac{\partial C}{\partial P_{i1}} = 4.169(0.177) P_{i1}^{(0.177-1)} P_{i2}^{0.215} P_{i3}^{0.149} P_{i4}^{0.114} P_{i5}^{0.152} P_{i6}^{0.080} P_{i7}^{0.052} P_{i8}^{0.061} \tilde{y}_i^{0.333}$

or, $x_{i1} = \frac{0.738\, P_{i2}^{0.215} P_{i3}^{0.149} P_{i4}^{0.114} P_{i5}^{0.152} P_{i6}^{0.080} P_{i7}^{0.052} P_{i8}^{0.061} \tilde{y}^{0.333}}{P_{i1}^{0.823}}$

where, i = 1, 2, 3, …, 100, number of farms.

Joypurhat

I. Stochastic Production Function:

$\text{Ln } y_i = 10.176 + 0.512 \ln x_{i1} + 0.448 \ln x_{i2} + 0.299 \ln x_{i3} + 0.482 \ln x_{i4} + 0.315 \ln x_{i5} + 0.108 \ln x_{i6} + 0.007 \ln x_{i7} + 0.009 \ln x_{i8}$

or, $y_i = 10.176\, x_{i1}^{0.512} x_{i2}^{0.448} x_{i3}^{0.299} x_{i4}^{0.482} x_{i5}^{0.315} x_{i6}^{0.108} x_{i7}^{0.007} x_{i8}^{0.009}$

where, i = 1, 2, 3, …, 100, number of farms.

II. Dual stochastic frontier cost function:

$C(P_{ik}, \tilde{y}_i) = 1.968 P_{i1}^{0.235} P_{i2}^{0.206} P_{i3}^{0.137} P_{i4}^{0.221} P_{i5}^{0.144} P_{i6}^{0.050} P_{i7}^{0.003} P_{i8}^{0.004} \tilde{y}_i^{0.459}$

where, i = 1, 2, 3, …, 100, number of farms.

III. Input demand function:

$x_{i1} = \frac{\partial C}{\partial P_{i1}} = 1.968(0.235) P_{i1}^{(0.235-1)} P_{i2}^{0.206} P_{i3}^{0.137} P_{i4}^{0.221} P_{i5}^{0.144} P_{i6}^{0.050} P_{i7}^{0.003} P_{i8}^{0.004} \tilde{y}_i^{0.459}$

or, $x_{i1} = \frac{0.462 P_{i2}^{0.206} P_{i3}^{0.137} P_{i4}^{0.221} P_{i5}^{0.144} P_{i6}^{0.050} P_{i7}^{0.003} P_{i8}^{0.004} \tilde{y}^{0.459}}{P_{i1}^{0.}}$

where, i = 1, 2, 3, …, 100, number of farms.

Dinajpu

I. Stochastic Production Function:

$\text{Ln } y_i = 12.894 + 0.758 \ln x_{i1} + 0.445 \ln x_{i2} + 0.267 \ln x_{i3} + 0.381 \ln x_{i4} + 0.356 \ln x_{i5} + 0.067 \ln x_{i6} + 0.074 \ln x_{i7} + 0.052 \ln x_{i8}$

or, $y_i = 12.894\ x_{i1}^{0.758}\ x_{i2}^{0.445}\ x_{i3}^{0.267}\ x_{i4}^{0.381}\ x_{i5}^{0.356}\ x_{i6}^{0.067}\ x_{i7}^{0.074}\ x_{i8}^{0.052}$,

where, i = 1, 2, 3, …, 50, number of farms.

II. Dual stochastic frontier cost function:

$C(P_{ik}, \tilde{y}_i) = 2.056 P_{i1}^{0.316}\ P_{i2}^{0.185}\ P_{i3}^{0.111}\ P_{i4}^{0.159}\ P_{i5}^{0.148}\ P_{i6}^{0.028}\ P_{i7}^{0.031}\ P_{i8}^{0.022}\ \tilde{y}_i^{0.417}$

where, i = 1, 2, 3, …, 50, number of farms.

III. Input demand function:

$x_{i1} = \frac{\partial C}{\partial P_{i1}} = 2.056(0.316) P_{i1}^{(0.316-1)}\ P_{i2}^{0.185}\ P_{i3}^{0.111}\ P_{i4}^{0.159}\ P_{i5}^{0.148}\ P_{i6}^{0.028}\ P_{i7}^{0.031}\ P_{i8}^{0.022}\ \tilde{y}_i^{0.417}$

or, $x_{i1} = \frac{0.650\ P_{i2}^{0.185}\ P_{i3}^{0.111}\ P_{i4}^{0.159}\ P_{i5}^{0.148}\ P_{i6}^{0.028}\ P_{i7}^{0.031}\ P_{i8}^{0.022}\ \tilde{y}^{0.417}}{P_{i1}^{0.684}}$,

where, i = 1, 2, 3, …, 50, number of farms.

Rangpur

I. Stochastic Production Function:

$\text{Ln } y_i = 6.445 + 0.662 \ln x_{i1} + 0.429 \ln x_{i2} + 0.365 \ln x_{i3} + 0.439 \ln x_{i4} + 0.330 \ln x_{i5} + 0.263 \ln x_{i6} + 0.215 \ln x_{i7} + 0.193 \ln x_{i8}$

or, $y_i = 6.445\ x_{i1}^{0.662}\ x_{i2}^{0.429}\ x_{i3}^{0.365}\ x_{i4}^{0.439}\ x_{i5}^{0.330}\ x_{i6}^{0.263}\ x_{i7}^{0.215}\ x_{i8}^{0.193}$

where, i = 1, 2, 3, …, 50, number of farms.

II. Dual stochastic frontier cost function:

$C(P_{ik}, \tilde{y}_i) = 3.907\ P_{i1}^{(0.229)}\ P_{i2}^{0.148}\ P_{i3}^{0.126}\ P_{i4}^{0.152}\ P_{i5}^{0.114}\ P_{i6}^{0.091}\ P_{i7}^{0.074}\ P_{i8}^{0.067}\ \tilde{y}_i^{0.345}$

where, i = 1, 2, 3, …, 50, number of farms.

III. Input demand function:

$x_{i1} = \frac{\partial C}{\partial P_{i1}} = 3.907(0.229) P_{i1}^{(0.229-1)}\ P_{i2}^{0.148}\ P_{i3}^{0.126}\ P_{i4}^{0.152}\ P_{i5}^{0.114}\ P_{i6}^{0.091}\ P_{i7}^{0.074}\ P_{i8}^{0.067}\ \tilde{y}_i^{0.345}$

or, $x_{i1} = \frac{0.895\ P_{i2}^{0.148}\ P_{i3}^{0.126}\ P_{i4}^{0.152}\ P_{i5}^{0.114}\ P_{i6}^{0.091}\ P_{i7}^{0.074}\ P_{i8}^{0.067}\ \tilde{y}^{0.345}}{P_{i1}^{0.771}}$,

where, i = 1, 2, 3, …, 50, number of farms.

Appendix 3. Factors Affecting Inefficiency in Potato Farming

	Brahmanbaria			Narsingdi			Munshigonj			Joypurhat			Dinajpur			Rangpur		
Factors	TE	AE	EE	TE	AE	EE	TE	AE	EE	TE	AE	EE	TE	AE	EE	TE	AE	EE
	Coefficients (t-ratios)			Coefficients (t-ratios)			Coefficients (t-ratios)			Coefficients (t-ratios)			Coefficients (t-ratios)			Coefficients (t-ratios)		
Constant	0.109 (10.012)	0.116 (4.229)	0.059 (3.012)	0.085 (8.158)	0.258 (2.309)	0.277 (2.263)	0.207 (7.625)	0.146 (9.88)	0.368 (2.490)	0.226 (15.108)	0.063 (3.302)	0.064 (5.275)	0.019 (3.001)	0.140 (8.889)	0.163 (3.605)	0.050 (4.767)	0.264 (2.353)	0.310 (2.236)
Age	-0.005 (-4.707)	-0.003 (-2.987)	-0.008 (-5.837)	-0.009 (-7.944)	-0.004 (-3.583)	-0.007 (-3.234)	-0.004 (-3766)	-0.002 (-2.259)	-0.004 (-3.514)	-0.002 (-2.141)	-0.009 (-4.673)	-0.009 (-4.696)	-0.003 (-2.701)	-0.008 (-4.803)	-0.003 (-2.838)	-0.004 (-3.608)	-0.005 (-2.219)	-0.005 (-2.579)
Education	-0.003 (-3.259)	-0.006 (-2.644)	-0.004 (-3.091)	-0.007 (-2.690)	-0.004 (-4.141)	-0.003 (-2.288)	-0.007 (-4.824)	-0.006 (-4.104)	-0.003 (-2.259)	-0.006 (-6.403)	-0.009 (-4.192)	-0.007 (-8.055)	-0.005 (-4.745)	-0.004 (-3.249)	-0.007 (-4.696)	-0.005 (-3.640)	-0.009 (-4.507)	-0.007 (-5.604)
Experience	-0.006 (-5.226)	-0.004 (-3.763)	-0.003 (-2.834)	-0.008 (-3.374)	-0.006 (-5.253)	-0.004 (-3.674)	-0.008 (-7.085)	-0.007 (-2.376)	-0.004 (-3.820)	-0.008 (-3.337)	-0.003 (-3.134)	-0.003 (-2.338)	-0.006 (-4.086)	-0.005 (-2.131)	-0.002 (-2.136)	-0.008 (-4.005)	-0.005 (-3.929)	-0.003 (-2.367)
Land Frag - mentation	0.054 (2.720)	0.041 (2.992)	0.065 (3.824)	0.018 (5.015)	0.011 (0.502)	0.017 (2.716)	0.001 (2.159)	0.003 (2.680)	0.006 (3.138)	0.003 (2.528)	0.017 (3.018)	0.017 (3.993)	2.937	0.004 (2.447)	0.007 (2.743)	0.015 (2.683)	0.031 (2.967)	0.023 (2.281)
Family size	0.002 (0.167)	0.009 (1.321)	0.010 (1.133)	0.002 (0.289)	0.008 (4.373)	0.002 (1.063)	0.008 (0.552)	0.017 (1.997)	0.009 (0.892)	0.005 (1.137)	0.011 (0.774)	0.126 (0.826)	0.015 (0.937)	0.002 (0.351)	0.010 (1.102)	0.005 (0.335)	0.004 (0.415)	0.006 (0.519)
Deweeding	-0.001 (-5.126)	-0.001 (-2.898)	-0.001 (3.466)	-0.002 (-2.011)	-0.001 (-2.166)	0.001 (-2.195)	-0.001 (-2.630)	-0.011 (-2.062)	-0.006 (-2.295)	-0.002 (-2.304)	0.002 (-2.876)	-0.001 (-2.815)	-0.002 (-6.213)	-0.002 (-2.160)	-0.001 (-2.217)	-0.015 (-5.976)	-0.001 (-2.702)	-0.011 (-2.145)
Access to Credit – Dummy							-0.153 (-8.116)	-0.008 (-5.428)	-0.076 (-5.340)	-0.015 (-11.320)	-0.014 (-3.089)	-0.009 (-4.207)				-0.189 (-3.624)	-0.105 (-9.624)	-0.038 (-3.782)
Cold Storage – dummy							-0.167 (-2.113)	-0.165 (-2.062)	-0.226 (-2.352)	-0.191 (-4.288)	-0.116 (-2.982)	-0.121 (-3.182)				-0.307 (-2.276)	-0.087 (-2.100)	-0.212 (-2.198)
Training – Dummy							-0.049 (-2.621)	-0.034 (-2.920)	-0.050 (-3.613)	-0.032 (-18.462)	-0.087 (-2.992)	-0.092 (-2.056)	-0.071 (-5.572)	-0.009 (-3.404)	-0.043 (-2.156)	-0.245 (-3.357)	-0.008 (-5.973)	-0.132 (-8.847)

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Covid-19: Epidemiology, Challenges and Strategies in the Context of Bangladesh

Md. Mobarak Hossain Khan*
Department of Social Relations, EWU, Aftabnagar, Dhaka, Bangladesh

Abstract

Covid-19 is one of the most devastating public health problems worldwide since World War II. The negative impacts of Covid-19 on human lives are numerous, which are generally more dominant in resource-poor developing countries than developed ones. Bangladesh is one of worst hit countries by this pandemic, which has been struggling from the beginning and is likely to face more difficulties in future. The major aims of this article are to provide some epidemiological information concerning Covid-19 and then explain some of the major challenges caused by Covid-19 focusing on Bangladesh. Secondary sources of data are used to prepare this article. The major sources of data are PubMed, Google Scholars, Websites of World Health Organization, Directorate General of Health Services, and Institute of Epidemiology, Disease Control and Research.

Covid-19 cases and deaths for the period of March 1, 2020 to November 22, 2021 showed remarkable fluctuations in Bangladesh. The highest peak for corona cases was seen in the period of July-August 2021, followed by April-May 2021 and June-July 2020 and December 2020-January 2021. Similarly, remarkable variations in deaths are observed in Bangladesh. The number of daily deaths was over 200 for most of the period from July 7 to August 12, 2021, although such deaths are relatively lower for other periods. As of December 8, 2021, the overall case fatality rate (CFR) was 1.8% and the recovery rate was about 97.8%. Available evidence highlighted various kind of challenges for Bangladesh. These can be broadly categorized as economic problems, healthcare mismanagement, nutritional deficiencies and social issues. Economic problems can be revealed by declining foreign remittance and household income, increasing rate of unemployment and poverty. Healthcare mismanagement is indicated by e.g., lack of Covid-19 dedicated beds in hospitals, inadequate service providers and testing facilities, and lack of regular treatment for co-morbid people. Lack of vaccination including high level of vaccine hesitancy are also important indicators of healthcare mismanagement. Nutritional deficiencies may include increasing rates of food insecurity and malnutrition (stunting, wasting, low BMI, and anemia). Social issues such as increasing incidence of domestic violence and crime, suicidal ideation, poor mental health, and social inequality are evident in Bangladesh. Disrupted educational system and violation of preventive measures by a large segment of the population are also worthy to mention for Bangladesh. In conclusion Covid-19 has already caused multidimensional challenges in Bangladesh. To minimize their current impacts and future consequences, the Government of Bangladesh including policymakers should urgently identify major weaknesses of the existing strategies and take more feasible actions for better outcomes.

Key Words: Covid-19, Pandemic, Epidemiology, Challenges, Bangladesh.

1. Background

Pandemics are nothing new and occurred regularly throughout human history. Some of the major pandemics such as plague, cholera, Spanish Flu, and HIV/AIDS including Covid-19 caused millions of deaths and severely afflicted the humanity (Sampath et al., 2021; Piret and Boivin, 2020). Covid-19 is one of the most severe pandemics in the human history, which was first reported as a pneumonia in

* Corresponding author: mmhkhan@ewubd.edu

December 2019 in the Wuhan city of Hubei Province of China. Initially the etiology of this disease was unknown. Later on January 7, 2020 it was identified as a novel coronavirus disease by the Chinese authority and shared its genetic sequence on January 12, 2020 for other countries to develop specific testing kits (see WHO situation report-1, published on January 22, 2020). It was caused by Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2), which could infect both human and animal (Islam et al., 2020; Al-Bari et al., 2021). Its transmission can occur through human-to-human via direct contact or via droplets produced by coughing, sneezing, deep breathing and talking within a proximity of infected person (Rume and Islam, 2020; Al-Bari et al., 2021). It can also be transmitted through touching contaminated surfaces (Hossain et al., 2021).

This disease has been highly contagious from the beginning and its consequences are found to be very complex and serious for both developed and developing countries. Every sphere of human lives is damaged by its attack (Rume and Islam, 2020; Rahman et al., 2021a). The severe cases of Covid-19 can lead to multiple problems such as cardiac injury, respiratory failure, acute respiratory distress and death (Rume and Islam, 2020). The rapid spread of the virus and its ravaging impacts and outcomes are also associated with the increased level of fear, panic, stigmatization, depression and anxiety among people around the world (Shammi et al., 2020; Asante et al., 2021; Hossain et al., 2021). Despite of various multilevel (local to global) non-pharmaceutical or non-therapeutic controlled measures implemented by the concerned authorities (Anwar et al., 2020; Asante et al., 2021), it has rapidly surged all over the world and has seriously threatened the global public health (Shammi et al., 2020).

Evidence regarding Covid-19 epidemiology and its consequences are obviously very critical for any country to minimize both current and future threats. Particularly, the Government of Bangladesh (GoB) and other stakeholders badly need such information to design and implement feasible policies and programmes for combatting its ongoing and future consequences. Considering the scarcity and importance of evidence-based public health recommendations, an attempt has been made to provide useful information regarding its epidemiology and major challenges including some feasible control strategies with a special reference to Bangladesh. The review article is divided into various sections and sub-sections. The main aim of the section two is to explain how Covid-19 had quickly spread from China to other countries and became a global public health concern within one-month period. In section three, epidemiological data of Covid-19 is presented for Bangladesh. In section four, a set of Covid-19 attributed challenges (current or future) is documented. Future strategies (short- and long-term) including concluding remarks are mentioned in the following sections.

This paper is purely based on the secondary sources of data, mainly from PubMed, Google Scholars, Newspaper articles, Websites of different organizations (e.g., WHO, Worldometer, IEDCR/DGHS), etc. All these sources are adequately acknowledged using citations and web-links. The figures and tables are made by the author.

2. Covid-19 from China to Other Countries

It is already mentioned that the origin of Covid-19 was the Hunan seafood market in Wuhan Province of China. The first case was reported on December 31, 2019. Table 1 shows the reporting dates of Covid-19 case(s) by countries affected within one month (until January 31, 2021) from its first occurrence. It also shows the distribution of cumulative and newly added cases by these countries. According to the information, Thailand was the 2nd country globally after China, which reported the first case of Covid-19 imported from Wuhan. Chronologically, other three countries, which reported Covid-19 cases, were Japan (3rd), Republic of Korea (4th) and United States of America (5th) (WHO, 2020; Situation Reports 1 and 2). Italy was the last (20th) country, which was affected within one-month period from its beginning. Sometimes, multiple countries had reported the first occurrence of Covid-19 on the same date. For example, Australia, Nepal and France had reported the first case on Jan 25, 2021.

Initially, total cases of Covid-19 in the affected countries had increased exponentially, mainly due to rapid spread in China. For instance, a total of 282 confirmed cases was reported on January 20, 2021 in four countries, which became 846 (3 times more) on Jan 24, 2021 in seven countries, and 2,798 (9.9 times more) on Jan 27, 2021 in 12 countries and 9,826 (34.8 times more) on Jan 31, 2021 in 20 countries. During that initial one-month period, most of the cases (over 98%) were reported in China.

Table 1. Countries Affected by Covid-19 within one Month (Dec 31, 2020 to Jan 31, 2021)*

Reporting date	Cumulative cases		Covid-19 countries	Newly added
	Global	China		-
Dec 31, 2019	-	-	China (first)	-
Jan 13, 2020	-	-	Thailand was newly added (second)	-
Jan 15, 2020	-	-	Japan was added (third)	-
Jan 20, 2020	282	278	Korea was newly added (fourth)	-
Jan 21, 2020	314	309	No country was newly affected	32
Jan 23, 2020	581	571	US was newly affected (fifth)	267
Jan 24, 2020	846	830	Vietnam and Singapore were newly affected	265
Jan 25, 2020	1320	1297	Australia, Nepal and France were newly affected	474
Jan 26, 2020	2014	1985	Malaysia was newly affected	694
Jan 27, 2020	2798	2761	Canada was newly affected	784
Jan 28, 2020	4593	4537	Cambodia, Sri Lanka, Germany	1795
Jan 29, 2020	6065	5997	UAE	1472
Jan 30, 2020	7818	7736	Philippines, India, Finland	1753
Jan 31, 2020	9826	9720	Italy	2008

*These data are collected from the WHO Novel Coronavirus (2019-nCov) Situation Reports - 1 (Jan 21, 2020) to 11 (Jan 31, 2020).
(Source:https://www.who.int/emergencies/diseases/novel-coronavirus-2019/situation-reports)

Since Covid-19 had continued its fast spread to many countries of the world, it was declared as a pandemic disease on March 11, 2020 by the World Health Organization (WHO) (Hossain et al., 2021). Already many cases including deaths are reported worldwide. According to the World meter data (https://www.worldometers.info/coronavirus/), globally around 258 million (i.e., 258,575,809) corona cases and 5.18 million (i.e., 5,178,467, case fatality rate = 2.00%) deaths were recorded on November 22, 2021. Approximately 234 million cases had recovered (i.e., 234,061,609, recovery rate = 90.52%) from this disease by the same date. (https://www.worldometers.info/coronavirus/).

According to the information of Table 2 (based on: https://covid19.who.int/table, accessed on Nov 23, 2021), 226 countries or other territories/areas of the world were affected by this disease. Among them, 40 countries had reported at least 1 million cases, varies from approx. 1.03 million (Hungary) to 47.37 million (USA) cases. Other four leading countries were India (34.52 million, 2nd), Brazil (22.01 million, 3rd), UK (9.85 million, 4th) and Russian Federation (9.37 million, 5th), respectively.

Table 2. Classification of Countries by Number of Covid-19 Cases (as of October 18, 2021)

Cumulative cases of Covid-19	Number of Countries/ Territories/Areas	Some selected countries
40 million+	1	USA (44.41 million, rank 1)
30-40 million	1	India (34.52 million, rank 2)
20-30 million	1	Brazil (22.01 million, rank 3)
10-20 million	0	
1 - 10 million	37	UK (9.85 million, 5th position), Russian Federation (9.37 million, 5th position), Argentina (5.32 million, 10th position), South Africa (2.93 million, 18th position), Bangladesh (1.57 million, 31th position), Hungary (1.03 million, 40th position)
Below 1 million	186	Nepal (0.82 million), Sri Lanka (0.56 million), Kuwait (0.41 million), Singapore (0.25 million), China (0.13 miliion), maldives (0.09 million), Samoa (only 1 case).
No case	11	Turkmenistan, Saint Helena, Kiribati, Cook Islands
Total	237	

Source: https://covid19.who.int/table (accessed on 23.11.2021)

3. Covid19 Situation in Bangladesh

Bangladesh is one of the badly affected countries by Covid-19 with 1.58 million cases (i.e., 1578288 cases) (as of December 8, 2021). Several factors such as high population density (Islam et al, 2020; Islam et al., 2020a; Kumar and Pinky, 2020; Hossain et al., 2021), poor healthcare infrastructures (Islam et al., 2020; Alam and Khatun, 2021; Hossain et al., 2021), widespread poverty (Islam et al., 2020; Hossain et al., 2021) and the weak economy (Islam et al., 2020; Islam et al., 2020a; Alam and Khatun, 2021) increased the risk of infection in Bangladesh. Inadequate number of well-equipped hospitals and testing facilities, lack of awareness and hygiene practice, tendency to violate prevention measures (e.g., lockdown) and the precarious employment aggravated the overall situation of Covid-19 in Bangladesh (Islam et al., 2020a). Moreover, other hazards such as cyclones, floods and outbreak of dengue along with poor and fragile healthcare system heightened the risks of Covid-19 in Bangladesh (Rahman et al., 2021).

Bangladesh got over 2 months to identify the first case of Covid-19 since its emergence in China. The Directorate General of Health Services (DGHS) had declared three cases of Covid-19 patients (male, foreign, etc.) on March 8, 2020 for the first time (Hossain et al., 2021; Rahman et al., 2021a). After this report, Bangladesh became very alert and implemented multiple control measures suggested by WHO. However, many of the containment measures were not successfully implemented in Bangladesh. General population in general and high-risk groups in particular were reluctant to follow such measures. For instance, overseas returnees or expatriates were unwilling to maintain quarantine for 14 days either at the designated camps or home (Hossain et al., 2021). Many of them also violated recommended guidelines and were roaming around. As a result, cumulative cases of Covid-19 had increased very quickly in Bangladesh (Fig. 1).

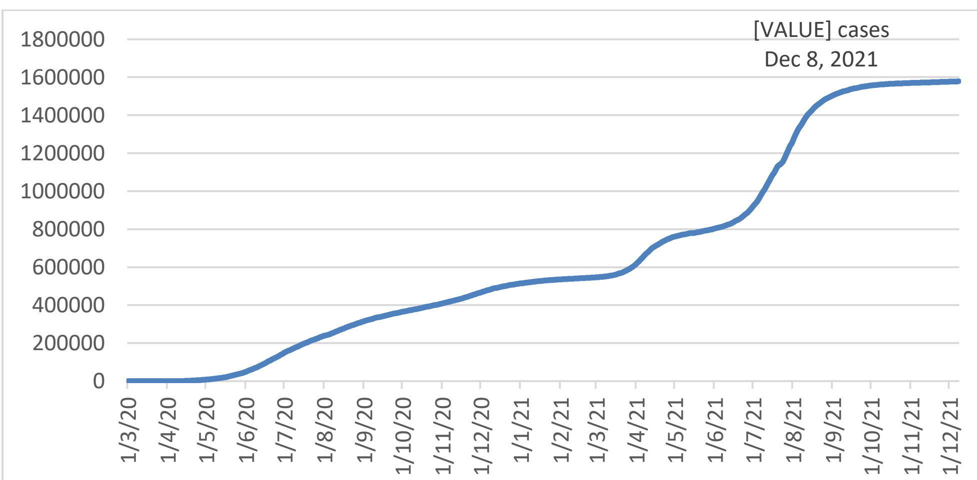

Figure 1. Cumulative Covid-19 cases in Bangladesh (March 1, 2020 to December 8, 2021)

The daily incidence of Covid-19 (Fig. 2) in Bangladesh also revealed several peaks during the period from 2020 to 2021. The highest peak was seen in the period of July-August 2021, followed by April-May 2021 and June-July 2020 and December 2020-January 2021. However, the most recent data of daily cases indicates some sorts of improvement in Bangladesh.

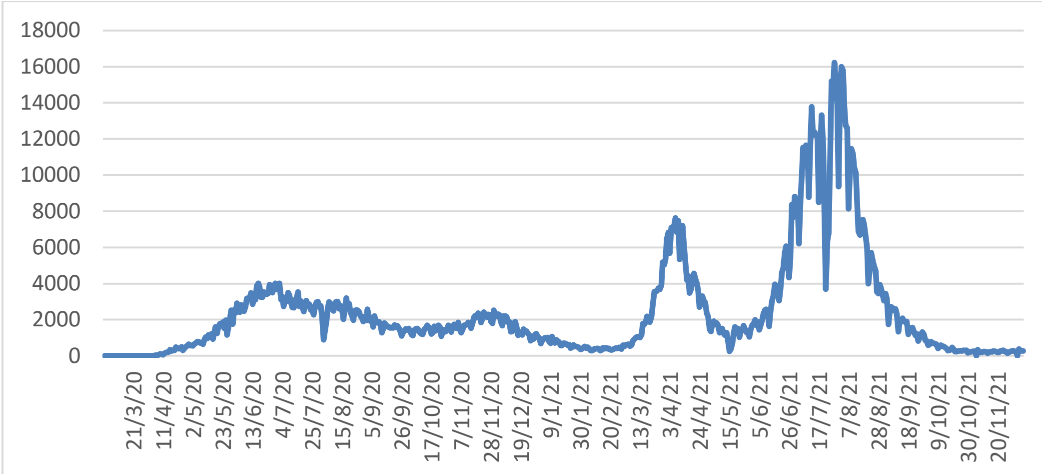

Figure 2. Covid-19 Daily Cases in Bangladesh (March 1, 2020 to December 8, 2021)

Like Covid-19 cases, cumulative death increased significantly in Bangladesh (Fig 3). We can also see several peaks of Covid-19 daily deaths in Bangladesh (Fig. 4). From the beginning to the middle of April 2021, the daily number of deaths were either below or around 50 (Fig. 4). Suddenly, it started to rise after this time, and had crossed the number 100 at the end of May 2021. Then it had gradually declined to the level of below 50 again and remained below 50 until July 2021. Daily deaths started to increase again and crossed the number 200 in mid-July 2021. Unfortunately, the number of daily deaths was over 200 for most of the period from July 7 to August 12, 2021. The highest number of deaths was 264, which were recorded on August 5 and 10. From August 28, 2021, the number of daily deaths became below 100 per day.

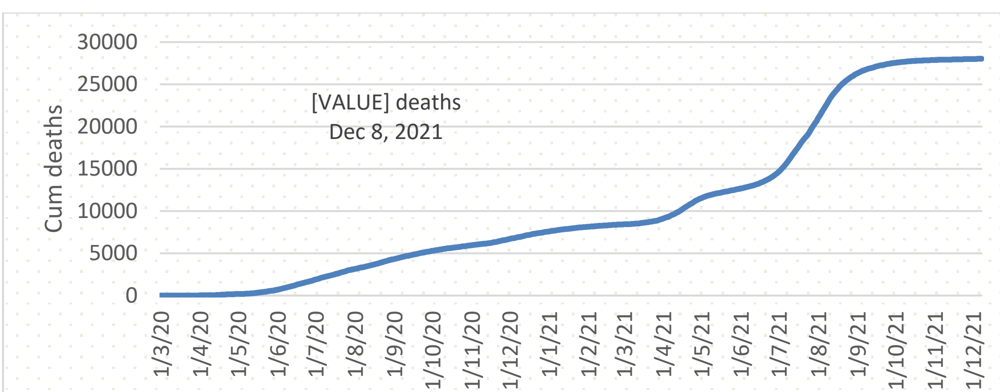

Figure 3. Cumulative Covid-19 Deaths in Bangladesh (March 1, 2020 to December 8, 2021)

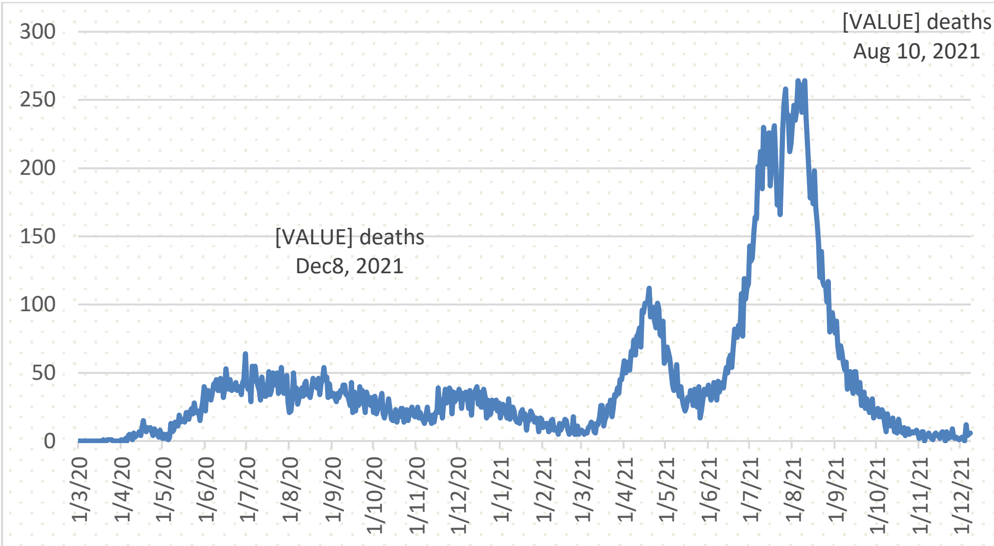

Figure 4. Daily Deaths of Covid-19 in Bangladesh (March 1, 2020 to December 8, 2021)

Fig. 5 is used to present the distribution of total Covid-19 cases (from the beginning to October 15, 2021) by 8 divisions in Bangladesh. Over 50% of the total cases are reported in Dhaka division, followed by Chittagong (18.1%), Khulna (7.5%) and Rajshahi (7.3%) divisions. The reported number of cases are below 3% for other divisions in Bangladesh.

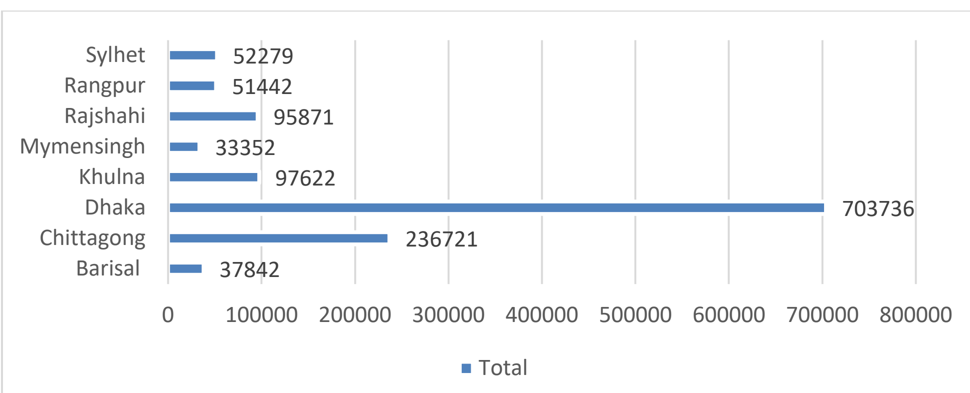

Figure 5. Distribution of Covid-19 Cases by 8 Divisions in Bangladesh

According to the data of Table 3, two districts with big cities (Dhaka and Chittagong) were the major hotspots for both periods of 2020 (Mar 1 to Dec 31, 2020) and 2021 (Jan 1 to Oct 15, 2021). Comparatively, the second period recorded more cases than first period. According to the data of second period, Comilla, Sylhet, Narayanganj and Gazipur districts were more affected in Bangladesh.

Table 3. Comparison of Covid-19 Affected top 10 Districts in Bangladesh between 2020 and 2021

From Mar 1, 2020 to Dec 31, 2021	Covid-19 cases	Rank of districts	From Jan 1, 2021 to Oct 15, 2021	Covid-19 cases	Rank of districts
Dhaka	1,89,025	1	Dhaka	3,35,114	1
Chittagong	24,216	2	Chittagong	75,162	2
Bogra	11,077	3	Comilla	27,721	3
Comilla	9,391	4	Sylhet	21,597	4
Faridpur	8,220	5	Narayanganj	21,369	5
Narayanganj	8,165	6	Gazipur	21,258	6
Sylhet	8,139	7	Khulna	17,516	7
Khulna	7,696	8	Noakhali	17,180	8
Gazipur	6,168	9	Cox's Bazar	16,915	9
Cox's Bazar	6,152	10	Rajshahi	16,407	10

Males were more affected than females (http://103.247.238.92/webportal/pages/covid19.php), perhaps because of their higher mobilities and exposures to social gathering and public places. Similarly, older people were more affected than younger people, possibly because of higher prevalence of co-morbidities (e.g., diabetes, heart disease, cancer) and weak immune system (Rume and Islam, 2020; Mistry et al., 2021). Difference between rural and urban areas were also remarkable, with higher burden in urban areas (Rahman et al., 2021b). For example, Dhaka was the epicenter of the Covid-19 infection, which recorded majority cases in Bangladesh (Rahman et al., 2021b). Some of the major factors which may explain higher infection rate in urban areas are crowding and difficulties to maintain social distance.

4. Covid-19 Challenges

Like many other countries, Bangladesh had implemented various measures to control this disease. But many weaknesses or problems related to these measures were recorded (Hossain et al., 2021). One of the initial problems for Bangladesh and many other countries was the absence of vaccines or antiviral drugs approved by WHO. Only non-therapeutic or non-pharmaceutical measures were available to control the spread of this virus (Anwar et al., 2020; Rahman et al., 2021a). These measures along with high level of panic have affected each sector and every citizen of Bangladesh. Global economy, human health, healthcare management and nutrition are drastically disrupted by its unprecedented burden. World-wide impacts are observed through travel and events cancellation, higher unemployment and disruption in food chain, academia, and healthcare facilities (Shrestha et al., 2020) in both developed and developing countries (Alam and Khatun, 2021; Asante et al., 2021). It also created numerous challenges for other sectors and Sustainable Development Goals (SDGs) (Shammi et al., 2020a; Bukari et al., 2021). In this section, a set of challenges (listed below) are described for Bangladesh, based on published literature.

4.1 Economic Challenges

4.1.1 Economic Crisis

Economic activities have been remarkably contracted around the globe due to Covid-19 pandemic and its containment measures. Consequently, the economic well-being and livelihood of millions of vulnerable populations worldwide, mainly living in developing (low- and middle-income) countries, are unevenly jeopardized (Egger et al., 2021; Hossain 2021; Yadav 2020). One recent study conducted in nine developing countries including Bangladesh found that household income has remarkably dropped during the Covid-19 crisis. Indian gross domestic product shrank by 23.9% during April-June quarter in 2020 (Yadav, 2000). Although various preventive measures were taken by households and the governments, these were not adequate to improve precarious living standards for many of the affected populations (Egger et al., 2021).

Bangladesh Stock Market and foreign trade were affected by Covid-19 (Alam et al., 2020). Economic threats are also manifested by increased poverty, food insecurity, and unemployment (Kumar and Pinky, 2020). Particularly, foreign remittance, which is the leading source of socio-economic development, has declined by 22% during the period of 2019-2020 (Matlin et al., 2021). The overall socioeconomic gains which Bangladesh has achieved through poverty reduction and household income over the last few decades are at risk of being eroded due to income losses of marginal people. Since millions of people are dependent on foreign remittance, overall livelihood of their families is also severely affected. Poverty also increased due to declining flow of remittance and income generation and family debt (Hossain, 2021).

Table 4. List of multidimensional challenges caused by Covid-19 pandemic

4.1 Economic challenges	
	4.1.1 Economic crisis
	4.1.2 Increased unemployment
	4.1.3 Increased poverty
4.2 Healthcare challenges	
	4.2.1 Healthcare deficiency and mismanagement
	4.2.2 Increased burden of co-morbidities
	4.2.3 Vaccine hesitancy
4.3 Nutritional challenges	
	4.3.1 Household food insecurity
	4.3.2 Disruption in food supply chain
	4.3.3 Increased malnutrition

4.4 Socio-cultural challenges
4.4.1 Social inequality
4.4.2 Disrupted education system
4.4.3 Violence/crime/suicide
4.4.4 Violation of prevention rules

4.1.2 Increased Unemployment

Bangladesh is a labor-surplus country. Between 10 and 13 million Bangladeshi overseas migrants, who are mostly unskilled or semi-skilled, are working in other countries particularly in the Middle East (around 60%) and Asia Pacific (11%) regions. A significant number of these overseas workers were facing Covid-19 attributed problems, which included economic uncertainty, shrinking job opportunities in the host countries, uncertainty in job continuation, uncertain future for returnee migrants and expiration of visas for aspirant migrants. Already many overseas migrants from Bangladesh became jobless since their employers terminated their jobs to adjust to the new environment created by Covid-19 (Shammi et al., 2020a).

The overall livelihood and economic conditions of a significant number domestic migrants, informal workers and their dependents were also severely disrupted by multiple impacts of Covid-19. These are e.g., joblessness due to job termination and narrowed job markets, limited cash flow to the family members due to wage-cut or loss of income sources for workers. Unfortunately, many people are also at risk of losing jobs. The estimated number of informal workers who lost their jobs were approximately 20 million (Hossain 2021).

The negative impacts of Covid-19 on business and manufacturing industries are also seen in many countries including Bangladesh (Nicola et al., 2020). Declining turnover and production, closure of small business, disruption in supply chain, amount of import and export goods, cancellation of orders and termination of jobs are some of the leading consequences of Covid-19. Formal and informal businesses including the conditions of informal workers in Bangladesh are also remarkably affected (Shammi et al., 2020a; Hossain 2021). Garment industries are closed due to limited order from the buyers or supply of raw materials imported from other countries. The overall demand of the market goods is also reduced sharply due to difficulties arising from the increased unemployment (Shammi et al., 2020a).

Since migrants (internal or overseas) are the major source of income for maintaining livelihood of left-behind family members in rural areas, many families are experiencing multiple problems due to Covid-19 attributed unemployment or reduced income of the family. Increasing poverty, deprivation, hunger, food insecurity, health inequality, social conflicts, and poor physical and mental health (Pérez-Escamilla et al., 2020; Shammi et al., 2020; Shammi et al., 2020a) are some of the negative effects created by the Covid-19 attributed unemployment (Pérez-Escamilla et al., 2020; Shammi et al., 2020; Egger et al., 2021; Hossain 2021). It is projected that unemployment status of Bangladesh will be more dreadful in post Covid period due to Covid-19 related control measures (Hossain 2021).

4.1.3 Increased Poverty

Poverty is the leading global agenda of SDGs, which is likely to upsurge in the coming years due to Covid-19 (Asante et al., 2021). Economic contractions and food supply chain disruptions due to Covid-19 and its responses led 95 million people to extreme poverty. Moreover, around 150 additional people will be pushed into extreme poverty in 2020 due to Covid-19 pandemic and its responses particularly in Asia and Africa (Osendarp et al., 2021).

Particularly, marginal people who have very small or no saving at all are the worst victims of Covid-19. Unfortunately, many people became jobless and many are at risk of losing jobs (Hossain 2021). Because of these phenomena, many poor became poorer due to the imposed Covid-19 prevention policies and their negative impacts on the lives of marginal population (Hossain 2021). Since the overall demand for manufactured goods (e.g., export-oriented ready-made garments) and services produced by agricultural and labor-intensive sectors (e.g., construction workers, transport workers, street vendors, rickshaw pullers, day laborers, restaurant workers, owners of small grocery shops) are sharply declined, the economy of the marginal people are seriously affected (Hossain 2021).

4.2 Healthcare Challenges

4.2.1 Healthcare Deficiency and Mismanagement

Weak governance and fragile healthcare system of Bangladesh during the Covid-19 period became obvious (Bodrud-Doza et al, 2020; Islam et al., 2020a; Shammi et al., 2020). Multiple healthcare-related weaknesses concerning both preventive and curative services are reported in Bangladesh. Some of these weaknesses were deficiency of surgical masks and sanitizers; scarcity of laboratory facilities for detecting Covid-19 at early stage and personal protective equipment (PPE) for service providers and lack of quarantine options at the hospitals. Excessive price of masks and sanitizers are also reported in Bangladesh (Alam et al., 2020; Anwar et al., 2020). Sharing of misinformation through mass media was also noticed (Bodrud-Doza et al., 2020). Healthcare mismanagement was also indicated by the fake reports for Covid-19 test (e.g., Regent hospital a private hospital in Dhaka) and refusal of healthcare professionals (e.g., doctors, nurses, and lab technicians etc.) to provide services (Anwar et al., 2020). Difficulties for getting admission into the hospitals and other Covid-19 dedicated facilities are reported widely. The number of ICU beds and ventilators to treat severe cases was very limited (Islam et al., 2020a). Screening facilities in the airports and land ports were highly inefficient from the beginning (Alam et al., 2020). Corruption of taking money from the travelers or passengers by the service providers are also reported. Expensive treatment of Covid-19 patients without health insurance coverage also indicates the weaknesses of healthcare. Front-line workers such as doctors, nurses and other healthcare professionals struggled a lot due to shortage of personal protective equipment and lack of preparedness (Islam et al., 2020a). Moreover, the amount of medical waste resulting from haphazard disposal of disinfectants, mask, and gloves has increased and endangers the physical environment remarkably (Rume and islam, 2020).

All these weaknesses of the healthcare system exacerbated the panic of the general public in Bangladesh (Bodrud-Doza et al., 2020; Shammi et al., 2020). Only about 35.5% of the study participants reported high confidence in the country’s healthcare system (Abedin et al., 2021). Similarly, over 50% of the 105 countries reported disruptions in antenatal services and over 70% reported disruptions in routine immunization. UNICEF also reported around 30% reduction in nutritional services including school feeding in the low-and middle-income countries (LMICs) (Osendarp et al., 2021).

4.2.2 Increasing Burden of Co-morbidities

Even before Covid-19, Bangladesh has to face challenges to provide services for communicable and non-communicable diseases. This country is considered as a double-burden country, because the overall burden from communicable diseases is high and the burden of non-communicable diseases is gradually increasing. The major causes are e.g., socio-economic development, rapid urbanization, increasing obesity, changes in lifestyles and availability of fast foods. Moreover, our society is aging gradually. Unfortunately, Covid-19 has affected the people with co-morbidities and hence increased the burden of non-communicable chronic diseases. For instance, a large percentage of older people with non-communicable diseases has reported difficulties in receiving routine medical care and accessing medicine in Bangladesh during this pandemic (Mistry et al., 2021). Movement of co-morbid patients outside home was restricted due to the lockdown. Moreover, healthcare facilities remained closed and service providers were reluctant during the lockdown. The abovementioned factors along with limited social activities and gathering increased the burden of mental health problems (e.g., poor mental well-being, stress, anxiety, domestic violence) among the co-morbid people and their family members. The burden of co-morbidity also increased due to changes in public health-related determinants (e.g., job, income, poverty, remittance, food, nutrition, stigmatization, discrimination, and health insurance) of the vulnerable population.

4.2.3 Vaccine Hesitancy

High rates of Covid-19 vaccination worldwide are imperative to acquire herd immunity and to minimize cases and deaths (Cascini et al., 2021). Bangladesh kicked off its coronavirus vaccination on January 27, 2021 for the frontline workers (26 people were vaccinated) and on February 7, 2021 (31,160 people were vaccinated) for the public (Abedin et al., 2021). As of December 8, 2021 (Fig. 6), the percentage of Bangladeshi people received at least one dose of vaccine was 63.22%, which was 39.38% for single dose and 22.84% for two doses (https://ourworldindata.org/covid-vaccinations?country=OWID_WRL). This data clearly indicates that we are far from the benchmark of "herd immunity", which can be achieved through either mass vaccination or infection. Since Covid-19 infection is not a good choice because of high case fatality rate and consequences, we should provide more emphasize on vaccination.

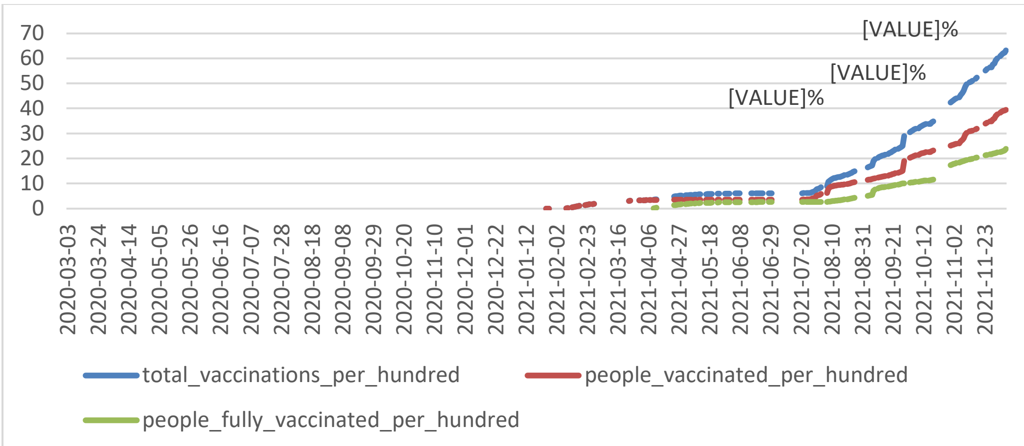

Figure 6. Covid-19 Vaccination Coverage in Bangladesh (as of November 28, 2021) (Source: Our World in Data)

Covid-19 vaccination in Bangladesh seems to be challenging among some groups of population (e.g., rural and slum communities), which is revealed by the high level of vaccine refusal and hesitancy. According to the study of Abedin et al (2021), about one-quarter of the study participants reported their unwillingness to vaccinate against Covid-19 (Abedin et al., 2021). App-based registration process for vaccination increased hesitancy particularly among people with low literacy level. The refusal rate was significantly higher among vulnerable groups such as elderly, rural and slum communities, farmers, day laborers, and low-educated people and among people with low confidence in healthcare system (Abedin et al., 2021).

4.3 Nutritional Challenges

4.3.1 Household Food Insecurity

Food insecurity was reported as a huge challenge in low- and middle-income countries including Bangladesh (Alam and Khatun, 2021; Pereira and Oliveira, 2020; Pérez-Escamilla et al., 2020; Bukari et al., 2021; Osendarp et al., 2021). It includes various types of food insecurity, ranging from mild to severe. In Bangladesh, mild to severe food insecurity was reported by approximately 90% of the households during Covid-19 (Das et al., 2020). Comparatively, urban households were more vulnerable than rural ones. For instance, severe food insecurity was mentioned by 42% urban and 15% rural households (Das et al., 2020).

4.3.2 Disruption in Food Supply Chain

Bangladesh is the 3rd leading countries in the world with respect to vegetable production. About 16.2 million farm households are cultivating vegetables, many of them are produced for commercial and export purposes. Unfortunately, the price of common vegetables (highly perishable) had dropped by more than half due to economic crisis and disruption in food supply chain. Generally, any shocks and stressors (e.g., lockdown and flood) can substantially disrupt different components of food supply chain, reduce access to healthy foods and increase the risk of household food insecurity and malnutrition (Osendarp et al., 2021; Pereira and Oliveira, 2020; Pérez-Escamilla et al., 2020; Alam and Khatun, 2021; Bidisha et al., 2021). Such losses may increase livelihood vulnerability, food insecurity and other challenges for many of the poor farmers (Alam and Khatun, 2021; Bidisha et al., 2021). These problems may also increase long-term developmental, psychological, physical and emotional harms to children of vulnerable (low-income) families (Das et al., 2020; Pérez-Escamilla et al., 2020; Alam and Khatun, 2021; Bidisha et al., 2021; Osendarp et al., 2021). For example, over 85% households reported anxiety for food supply in Bangladesh (Das et al., 2020). Like Bangladesh, over 50% of the sampled households did not have enough income, food, clean drinking water and proper medical treatments during the pandemic in Ghana (Bukari et al., 2021). Comparatively, all the nutritional challenges are more common in those low- and middle-countries, which lack political, economic and social prevention strategies for ensuring jobs (Das et al., 2020; Pérez-Escamilla et al., 2020; Alam and Khatun, 2021; Bidisha et al., 2021; Osendarp et al., 2021).

4.3.3 Increased Malnutrition

Various forms of malnutrition and deaths caused by Covid-19 are also reported in many countries including Bangladesh (Alam and Khatun, 2021; Pereira and Oliveira, 2020; Pérez-Escamilla et al., 2020; Bukari et al., 2021; Osendarp et al., 2021). Osendarp et al (2021) recently estimated additional number of children under five years of age, who might suffer from various kinds of malnutrition by 2020. According to moderate estimates, such additional children could be 9.3 million (min=6.4, max=13.6) for wasting, 2.6 million (min=1.5, max=3.6) for stunting and 2.1 million for low BMI at birth. Most of them are from South Asia and sub-Saharan Africa. Similarly, additional 2.1 million maternal anemia cases and 168,000 child deaths are estimated for the same period. The productivity loss due to additional stunting and child mortality are estimated to be around US$29.7 billion. Moreover, annually US$1.2 billion could be required to mitigate these effects by scaling up nutrition interventions (Osendarp et al., 2021).

Generally, food insecure households apply various options to cope with food scarcity (Das et al., 2020; Pereira and Oliveira, 2020; Osendarp et al., 2021. For instance, around 70% of the landless agricultural households in Bangladesh skipped meals or reduced the amount of food to cope with the crisis (Egger et al., 2021). About 90% of the households took less quality/nutritious food and 60% consumed less amount of food (Das et al., 2020).

4.4 Socio-Cultural Challenges

4.4.1 Social Inequality

The multidimensional impacts of Covid-19 on human lives (expressed as morbidity, mortality, well-being, livelihood) are far reaching and are disproportionately distributed across geographical locations and the sociodemographic strata (Asante et al., 2021; Egger et al., 2021). Its impacts are expected to be more on resource-poor countries such as Bangladesh (Bidisha et al., 2021) and Ghana (Asante et al., 2021) than rich countries (Egger et al., 2021). Generally, economic losses of developed countries are mitigated through social safety net programs of the government, employers' policies and household savings (Egger et al., 2021). Due to insufficient protection options, poor become poorer and more vulnerable due to Covid-19 in developing countries like Bangladesh (Bidisha et al., 2021).

Covid-19 has increased social inequality in terms of e.g., food insecurity (Bidisha et al., 2021) and healthcare access (Asante et al., 2021). For example, lower income or unprivileged groups such as day laborer and informal workers are suffering more from Covid-19 (Asante et al., 2021). Food insecurity is higher among unprivileged groups as the share of the food consumption expenditure is higher than higher income people in Bangladesh. Similarly, people living in environmentally endangered areas (such as climate prone areas) are more affected by the impact of Covid-19 (Bidisha et al., 2021). The severity of Covid-19 including fatalities are disproportionately higher among older people, people with co-morbidities, and males (Al-Bari et al., 2021). Some studies reported that male sex hormones are immunosuppressants, whereas female sex hormones enhance immunity (Al-Bari et al., 2021). Moreover, rural areas are not adequately equipped to control Covid-19 infection. Low level of awareness and practice of preventive behaviors, risk communication, isolation centers, and critical care treatment are limited in rural areas (Rahman et al., 2021b). Some of the social hazards such as job losses, lack of income and savings, lack of welfare support, lack of economic diversification and food insecurity are found to be associated with social inequality (Asante et al., 2021).

4.4.2 Disrupted Education System

The education system from pre-school to tertiary levels has been affected by Covid-19 and around 900 million learners are affected by the closure of the educational institutions. Adopted policies are found to be varied from country to country. Some countries (e.g., Germany) introduced complete closure and some countries implemented targeted closure (e.g., UK) to control spread of the virus (Nicola et al., 2020). Like many other countries, the Government of Bangladesh (GoB) also declared closure of all educational institutions (such as schools, college, and universities) on March 18, 2020 to avoid public gathering through staying at home (Alam et al., 2021; Rahman et al., 2021a). However, staying long period at home for young group of students is generally distressing because their regular lifestyles are disrupted in many ways. They may suffer from so called “cabin fever”, which means they may experience various psychological problems such as fear of exceeding the age limit of job, anxiety, impatience, frustration, restlessness, and symptoms of post-traumatic stress due to lockdown (Alam et al., 2021).

Such closures for an extended period of time are also distressing and problematic for parents of the students and their societies, particularly who are socioeconomically more vulnerable. Many parents of young children could not go to work and hence they lost their jobs and face economic insolvency. Social mobility and social isolation are decreased and school dropout rates are increased. Lower income people were unable to utilize technology-based virtual platforms to continue education during social isolation. It increased the economic burden due to expense of purchasing internet, computer, smart phones or other electronic devices. Non-covid related research are less prioritized and scientific conferences, networking opportunities for collaboration are cancelled or postponed (Nicola et al., 2020). Transformation from traditional system of physical classes and examinations to virtual ones was new and stressful for both teachers and students. Moreover, not all institutions successfully transformed this system.

4.4.3 Violence/ Crime/ Suicide

Since livelihoods of many families are severely interrupted due to lockdown and other preventive measures during this pandemic, several problems such as domestic violence, gender-based discrimination and crime has increased (Islam et al., 2020a). The high-level contagiousness of the Covid-19 and problems like stigmatization, social isolation, quarantine, depression, psychological trauma, substance abuse and other sequalae even leads to suicide (Shammi et al., 2020; Hossain et al., 2021). According to the nationwide survey results (Mamun et al., 2021), the rates of depression and suicidal ideation were 33% and 5%, respectively. The people of Dhaka city and adjacent districts, younger groups, females, smokers, people suffering from comorbidity, insomnia and higher level of fear reported higher level of depression and suicidal ideation in Bangladesh (Mamun et al., 2021). Similarly, the rates of suicide ideation and planning among Bangladeshi people were 19.0% and 18.5%, respectively. The suicidal risk was reported by 33.5% of the respondents, which are associated with female sex, divorced/widowed status, low educational attainment, Covid-19 prevalent areas, economic loss due to Covid-19, Covid-death in close relatives/acquaintances, direct contact with Covid patients, and fear of Covid-19 infection (Rahman et al., 2021a).

4.4.4 Violation of Preventive Rules

A large section of the society, particularly marginal and urban people such as day labors, garments workers and slum dwellers, are found to be very reluctant to abide by guidelines and preventive measures of the existing policies (e.g., wearing of masks, lockdown, social distancing). This can happen not only due to lack of knowledge and awareness but also due to economic (e.g., poverty, hunger), social/cultural (e.g., festival) and political (e.g., fear of losing jobs) environment of Bangladesh (Islam et al., 2020a). Around 16.8% of the study participants in Bangladesh reported adequate adherence to health safety regulations (Abedin et al., 20221). Violation of prevention rules at the time of religious festivals (e.g., Eid-ul-Adha) are also obvious in Bangladesh. Generally, these festivals are associated with the mass mobility of the general public in our country. Mass mobility is also a risk factor for infection. A large number of internal migrants from major cities, who came to these cities temporarily for better opportunities, go to their peripheral areas of Bangladesh to celebrate Eid with their remaining family members and close relatives. For example, about two-thirds of the Dhaka city population travel to their home towns or villages during festivals (Rahman et al., 2021b). According to the study of Rahman et al (2021b), the Covid-19 mean test positivity rate had increased from 9.5% (before the festival week) to 13% (first week after festival) and 17% (2nd week after festival). The infection rate in Mymensingh became two-folds within the next two weeks of festival as compared to the rate of infection on the day of festival (Rahman et al., 2021b).

5. Potential Strategies for Better Future

Prevention and proper management of cases are major options to control this disease (Shammi et al., 2020), because Covid-19 vaccination for all citizens and its mdeical treatment are still not available in Bangladesh. Like many other countries, the Government of Bangladesh (GoB) had implemented various measures (particularly non-therapeutic ones) to control the number of Covid-19 cases and deaths (Anwar et al., 2020; Islam et al., 2020). According to the approach of evidence-based public health, there are three important ways to implement prevention measures called information, incentives and obligations. The main aim of disseminating information is to create people’s knowledge and awareness about disease etiology and prevention strategies. In Bangladesh, electronic (e.g., TV), print (e.g., Newspapers) and social (e.g., Facebook) media have been very active to disseminate such information (Islam et al., 2020a). The DGHS is also providing daily report of Covid-19 and advising people to use mask and sanitizer during movement, maintain social distance or avoid social gathering. Isolation (for patients) and quarantine were also encouraged throughout the country to contain this disease.

Incentives (e.g., food, money) from both Government and Non-Government Organizations (NGOs) are also provided to the needy groups suffering from insufficient income and limited livelihood opportunities. In addition to GoB incentives, many affluent people came forward to help these disadvantaged people. Bangladesh had also enacted "nationwide lockdown measures" (as obligation) for all educational institutions, government and private offices and industries on March 26, 2020 to control the number of cases (Rahman et al, 2021a). These measures included travel bans, local and regional lockdown, social distancing, and remote office activities (Anwar et al., 2020). Moreover, the GoB deployed armed forces from March 24, 2020 to facilitate lockdown and other prevention policies in Bangladesh (Shammi et al., 2020). People were instructed to stay home during the lockdown. Persons who violated lockdown got varying levels of punishment by the concerned authorities.

It is already evident that implemented strategies in Bangladesh were not very optimal to contain the disease. Many challenges are newly created due to Covid-19 and had aggravated pre-existing socio-demographic, economic and healthcare challenges of the country. Keeping both current and anticipated consequences of the country in mind, some more strategies are mentioned below. Overall economic threats became evident in our country. In order to improve economic condition of the affected people, some recovery programs especially for the poor and marginal people are needed (Islam et al., 2020a). Adequate reliefs (both cash and kind) for the vulnerable groups are also important (Shammi et al., 2020). The GoB should emphasize on the employment-oriented economic policies that can create more jobs and reduce poverty and social inequality (Hossain 2021). Collaborative efforts during and after the post-pandemic periods are also necessary to help Covid-affected poor households (Asante et al., 2021).

Food insecurity is appeared as a challenge in Bangladesh. Multi-sectoral and inter-ministerial coordination and management mechanisms should be developed to ensure food and nutrition security for vulnerable groups (Pérez-Escamilla et al., 2020) and to reduce mental and social stress (Shammi et al., 2020). The GoB should provide more "cash support" than "food support" to ensure vegetable farming by the vulnerable farmers, so that better supply of food and vegetables can be maintained against the food shortage (Alam and Khatun, 2021). Healthcare deficiency and mismanagement during Covd-19 was another big challenge in Bangladesh. Since prevention and management of cases are the most feasible options to control this disease, certain improvements of the fragile healthcare system are necessary in Bangladesh (Shammi et al., 2020). More specifically, healthcare infrastructure such as laboratory facilities for the early detection of disease should be improved at all the divisional, district and Upazila hospitals. Isolation facilities should be improved to immediately isolate the Covid-19 case (Alam et al., 2020). Better coordination between stakeholders particularly between policy makers and healthcare providers are absolutely essential (Islam et al., 2020a). Concerted efforts (e.g., training of healthcare personnel, better management of hospital infections) are essential to minimize the weaknesses of the public health and healthcare systems in Bangladesh (Islam et al., 2020a). Improved medical waste management through incineration and management of other infections such as dengue are also important (Shammi et al., 2020).

The overall tendency to violate the preventive measures including lockdown was also common among general people. Therefore, stricter and more coordinated measures should be taken from local to national levels to ensure compliance with preventive measures (e.g., during the pandemic period) (Islam et al., 2020a). Particularly, proper responses with sound strategic planning and enforcement of existing laws are crucial for improving compliance in Bangladesh (Shammi et al., 2020a). Travel restrictions during festivals should be implemented more effectively (Rahman et al., 2021b).

Violence and crime have increased remarkably during the Covid-19 period due to e.g., limited livelihood and income facilities. Therefore, the GoB should be more active to provide effective services and safeguards to the victims of domestic violence and gender-based discriminations (Islam et al., 2020a). Human right to adequate foods and other basic needs should be guaranteed by the public policies particularly for the vulnerable populations (Pereira and Oliveira, 2020).

Available literature suggested higher burden of mental health problems in Bangladesh during Covid-19 pandemic period. Until recently, mental health issues are heavily neglected in our country. In order to improve this situation, concerned authorities should pay more and more attention to design and implement suitable prevention strategies during and after the Covid-19 pandemic (Rahman et al., 2021a). Location-based psychological support measures and improved access to mental health services, especially for women and young people, are necessary to minimize the burden of psychological problems in Bangladesh (Mamun et al., 2021). Vaccine hesitancy was common in Bangladesh. In order to increase vaccine coverage among general people, more policy-level initiatives including evidence-based mass media promotion are necessary. Public health professionals and other stakeholders must be ready to build vaccine literacy through evidence-based educational and policy-level interventions (Haque et al., 2021; Ali and Hossain, 2021). Mass media should be more active to disseminate vaccine-related information and eliminate the social stigma about the vaccination (Abedin et al., 2021). Moreover, community health workers, leaders and non-government organizations should be utilized to motivate people for vaccination by explaining its long-term positive effects (Ali and Hossain, 2021).

Bangladesh should strive to vaccinate at least 80 percent of the population as soon as possible, so that "herd immunity" can be achieved. For this purpose, Covid-19 vaccines should be available for all people at outreach centers (Abedin et al., 2021; Ali and Hossain, 2021; Haque et al., 2021). Collaborative research should be carried out to develop Covid-19 vaccines (Alam et al., 2020). More and more funds should be available for conducting scientifically sound research on Covid-19 epidemiology, surveillance and management in Bangladesh (Alam et al., 2020). Moreover, tracing of Covid-19 cases and their contacts should be improved throughout the country (Anwar et al., 2020). Testing facilities should be expanded and more and more kits should be available (Anwar et al., 2020). Like many other countries education sector in Bangladesh has been seriously affected by Covid-19 and its lockdown. Its consequences are huge and long lasting. Mental health problems e.g., psychological trauma, anxiety, and child abuse have been very common among students (BRAC, 2020). To recover the overall loss incurred in the education sector due to Covid-19 lockdown, various kinds of short, medium and long-term interventions should be implemented. The short-term interventions may include: strengthening the online learning process, providing alternatives to reduce technological barriers, broadcasting more classes and student-friendly recreational programmes, providing recorded classes and videos, reaching out the students at home on a regular basis and giving them proper advice, introducing hotline services to solve students' problems, etc. The medium-term strategies may include: providing incentives to the teachers for extra load and student's care, shortening the syllabus, and incorporating chapters to provide basic information regarding covid-19 and similar pandemics. The long-term strategy can include more budgets for each educational institution to equip them with required technologies and infrastructures, distant teaching-learning trainings for teachers, and a comprehensive and coordinated action plan for future (BARC, 2020).

To minimize the consequences of co-morbid people in Bangladesh, appropriate healthcare delivery pathways and strategies for the older people should be developed to ensure essential health services during any emergencies and beyond (Mistry et al., 2021). Moreover, emergency responses (both short and long term) based on the well-designed strategic plan should be formulated to minimize e.g., unemployment, poverty, food insecurity, child marriage, and domestic violence. Since data scarcity is a big challenge and hindering detailed analysis, multisectoral and collaborative studies, mainly longitudinal ones based on sound methodology and large sample size, should be started as early as possible to estimate the effects of Covid-19 on various outcomes such as poverty, unemployment, trade, business, tourism, education, food security, co-morbidity, crime and violence. Scientific evaluation of the already implemented or ongoing interventions should be done to find their weaknesses. The last but not least strategy could be the resilience building using proactive planning and implementation that helps to minimize multi-hazard threats in Bangladesh (Rahman et al., 2021).

6. Concluding Remarks

Covid-19 is a global public health problem and Bangladesh is one of the worst hit countries in the world. Although it appeared as a single public health problem in China, soon it became a strong determinant of many public health problems (e.g., depression, co-morbidities, malnutrition) everywhere. Unfortunately, a large number of the public health determinants such as poverty, unemployment, food insecurity, inequality, crime, violence, and socio-cultural values are seriously disrupted by the impacts of Covid-19. As a result, gained socio-economic development and other achievements of Bangladesh in the past are at serious stake. It should be noted that Covid-19 threats are heterogeneous and vary by social classes. Marginal and unskilled groups are suffering more by this pandemic disease. Informal sectors, particularly in big and divisional cities, are seriously affected as these sectors absorb most the unskilled labors and rural-urban migrants. Their income levels have sharply declined due to losing jobs or cutting salary. Until herd immunity through vaccination is achieved, public awareness and precautions are the most crucial keys to control this disease. Therefore, general public should follow recommended guidelines such as washing hands using sanitizers or soap for at least 20 seconds, avoiding mass gathering/meeting, using disposable tissues while coughing and sneezing. Moreover, they should seek immediate medical care and maintain self-quarantine when Covid-19 symptoms are present. They should also avoid public transport during epidemic period, avoid interpersonal contact, and contact hands with eyes, nose and mouth (Alam et al., 2020). Since it is a difficult task for the GoB to tackle this problem alone, other stakeholders such as mass media, political and religious leaders, civil society, social activists, top business organizations and international funders should come forward to create awareness among the mass population (Anwar et al., 2020).

We should not forest the positive effects of Covid-19 on society. Particularly, the overall level of knowledge, awareness and practice related to infection controls has been increased. Due to the slowdown of economic activities, limited transportation, international travel bans and other containment measures, the emission of greenhouse gases (GHGs), water and noise pollution, and the pressure on tourist destinations have been reduced sharply. As a result, air quality has significantly improved particularly in cities across the world. Moreover, such changes assist the restoration of the ecological systems (Rume and Islam, 2020).

In brief, Covid-19 is most likely to remain in our society for a long time and our future will be more complex due to its negative impacts. It might be more difficult to continue the same level of performance in the socio-economic and development sectors which we have achieved in the past few decades. A well designed and coordinated comprehensive action plan can help regain the Covid-19 losses of the country.

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Evaluating the Role of Information Technology Determinants on State-Owned Commercial Banks' Efficiency with Stochastic Frontier Models

Shakera Begum
Masters Student, Department of Statistics, The University of Akron, USA

Md. Azizul Baten*
Department of Statistics, School of Physical Sciences, Shahjajal University of Science and Technology, Sylhet-3114, Bangladesh

Rahmat Ali
Department of Statistics, School of Physical Sciences, Shahjajal University of Science and Technology, Sylhet-3114, Bangladesh

Abstract

This study developed an appropriate stochastic frontier model in the context of both cost and profit efficiency and evaluated the impact of Information Technology (IT) factors on State-owned Commercial Banks (SOCBs) in Bangladesh. Stochastic Cobb-Douglas frontier model, Translog stochastic frontier model and Tobit regression model are employed here considering a panel data set during 2007-2018. The average cost efficiency estimates of Cobb-Douglas for SOCBs was 59.2% whereas the average profit efficiency was 27.6% during the study period. While the average cost efficiency estimates of Translog was 81.3% and the average profit efficiency was 84.4% for SOCBs. Translog stochastic frontier model was found more preferable than Cobb-Douglas stochastic frontier model using Likelihood Ratio Test. The results of maximum likelihood estimates of both Stochastic Translog cost and profit frontier models show that bank loan was highly significant. The price of fund and price of labor and were significant and negative for the cost model. In Translog profit model, the majority of the variables were significant for the profit of SOCBs. The off-balance sheet items and the price of fund were positive with significant. The non- interest income was highly significant for the cost inefficiency model suggests that non- interest income did not impact banks cost. Again, non-performing loan, return on assets and capital adequacy ratio were negatively significant indication that they could be importantly influenced on banks profit efficiency. Both cost and profit efficiency were fluctuated over the study period and the cost efficiency scores was found higher than the profit efficiency during the years. The Janata bank was the most efficient among the other banks in case of both cost and profit. IT personnel expanses was positively significant in profit efficiency while in Translog model, IT personnel and IT personnel expanses had a negative impact on both cost and profit efficiency of SOCBs. This study is expected to be the first empirical study on both cost and profit efficiency of SOCBs; therefore, it shed a light on literature for the new aspect of bank efficiency.

Keywords: Efficiency, Stochastic frontier models, State-owned Commercial Banks, Information Technology, Bangladesh.

1. Introduction

Information technology is becoming more affordable now a days and the online banking sector is a complementary part of the banking sector as a whole. It is a new dimension to the banking sector to create

* Corresponding author: baten-sta@sust.edu

many opportunities for marketing products as well as managing organizations both domestically and internationally. The environment of internet banking requires authentication procedures for the electronic payment system, network environment, computer hardware and software, electronic hardware, legal bindings, etc. With the advancement of technology, the business process around the globe is eventually becoming complex where e-business in the banking sector, can supersede the conventional business process. With the expansion of global Information and Communication Technology (ICT) infrastructure and the internet, e-banking play a pivotal role in the national economic development of any country. But appropriate software, technology, infrastructure, skilled manpower, and Cyber law are crucial for implementing e-banking in the country. Ramakrishnan (2001) found that banking increases information security risks and have not sufficiently focused on the effect on other banking portfolio risks.

The function of banking has changed radically in developed countries and Bangladesh seem to increasingly adopt ICT banking and improving banking capabilities. Banking has the potential of changing the retail payments arena in a way that has not happened since the advent of credit card which helped to convert the paper-based payments to electronically-based payments which are more convenient to the people. However increasing competition among banks leads to losing their customers, but information technology by facilitating, service definition and new product and increasing efficiency at all levels of banking industry value chain, not only reduce the risk but create quality competitive advantages. ICT investments promote the enterprises' operational performance by reducing costs, raising profit margin, upgrading production levels, increasing service quality, advancing customer satisfaction and improving overall operations (Weill, 1992; Loveman, 1994; Mitra & Chaya, 1996; Devraj & Kohli, 2000). In contrast, other researchers did not demonstrate the positive effect of ICT investments and concluded that ICT spending brought no significant contributions to the enterprises' operations, and so the "ICT productivity paradox" has been an issue of continuous debate for decades (Strassman, 1990; Bryn Jolfsson,1993). Some researchers have found that only Stochastic Frontier Analysis (SFA) for the output and the value of ICT has made the studies on justifying ICT investments had a great impact on the bank efficiency but they did not use all the variables related to ICT (Romdhane, 2013; Rai & Patnayakuni, 1997; Surulivel, Vijayabanu, Amudha & Charumathi, 2013; Safari & Yu, 2014). Similarly, there is some researcher used both SFA & Data Envelopment Analysis (DEA) to analyze the impact on both cost and profit efficiency but unable to measure the whole ICT variables (Lee & Menon, 2000). On the other hand, in Bangladesh there are some studies on the cost and profit efficiency of banking by using SFA and DEA (Hasan & Hasan, 2018) but there have been no studies about the impact of ICT in the banking sector on the cost and profit efficiency.

The study has been undertaken to observe the present status of cost and profit efficiency of SOCBs identifying the ICT factor implementation. The country can be benefited through the successful development of ICT in banking and this study will help to enhance productivity and positive impact on raising economic growth too. So, this study measures the role of ICT on both cost and profit efficiency of SOCBs with both Stochastic Cobb-Douglas and Translog models in Bangladesh. The impact of IT components on both cost and profit efficiency of SOCBs using Tobit regression model is also investigated here.

2. Methodology

2.1 Data Sources

The data set used in this study obtained from the annual reports of banks during the period 2008-2017. The link of the annual reports the samples of SOCBs in Bangladesh are given below.

https://www.sonalibank.com.bd/PDF_file/annualreport/2018
https://www.jb.com.bd/about_us/annual_report
https://www.rupalibank.org

In this study all the data are yearly data and collected by two categories such as Non-IT data and IT data.

2.1.1 Definition or Description of the Variables

Table 1. Description of the Variables Including both IT and non IT

Variable Name	Variable Description
Dependent Variables	
Total Profit	It is a deduction of the total cost from total income. It takes after the tax.
Total Cost	Total costs include the income paid to depositor, personnel expenses, and other operating expenses.
Output Quantities	
Loan	The sum of long-term and short term loan.
Off-balance Sheet Items	Off-balance Sheet Items is measured as the sum of guarantees, commitment and financial derivative instrument.
Input Prices	
Price of Fund	Price of Fund is defined as the ratio of total interest expenses to all deposits.
Price of Fixed Assets	Price of fixed assets is measured as the non- interest expenses divided by fixed assets.
Price of Labor	Price of labor is calculated as the ratio of personnel expenses to the number of staff.
Explanatory Variables	
Non-interest Income	Non-interest income is a bank and creditor’s income derived primarily from fees including deposit and transaction fees, insufficient fund fees, annual fees, monthly account service charges, inactivity fees, check and deposit slip fees and so on.
Non-performing Loan	A non-performing loan is a sum of borrowed money upon which the debtor has not made the scheduled payments for a specified period.

Return on Assets	Return on assets is the ratio of annual net income to total assets of a bank during a financial year. Return on Equity: Return on equity is defined as the net profit divided by the shareholder average equity.
Capital Adequacy Ratio	The capital adequacy ratio equals the sum of the bank's tier one capital plus tier two capital divided by its risk-weighted assets.
IT Variables	
IT Expanses	The total IT expanses refers to incurred expanses for maintenance and repair, rent, depletion of IT equipment and information sourcing services.
IT Income	The total income from ICT Sector in bank.
IT Investment	IT investment is the total IT budget of the bank which included hardware, software, network, security training and other IT purpose.
IT Personnel	The total number of IT staff members in the bank.
IT Personnel Expenses	Price of labor is calculated as total salaries and staff expenses over full time number of staff.
ATM Card Transaction	The total amount of deposit withdraw by ATM Card.
ATM Card Expenses	The conduct of Banking Service Charge by using ATM Card. People can deposit their money in a bank account and they have entitled withdraw their money through ATM card.
Credit Card Transaction	The total amount of deposit withdraw by Credit Card.
Credit Card Expenses	Credit card is used for repayment of the value of products and services. This service charge is calculated price of credit card.

The measurement of both IT and non IT variables are described in Table 1.

2.2 Empirical Models

2.2.1 Empirical Cobb-Douglas Stochastic Frontier Cost Model

The specification of Cobb-Douglas stochastic frontier cost model (Battese & Coelli, 1995) is defined as:

$$\ln Y_{it} = \beta_0 + \beta_1 LOA_{it} + \beta_2 OBS_{it} + \beta_3 POF_{it} + \beta_4 POFA_{it} + \beta_5 POL_{it} + v_{it} + u_{it} \quad (1)$$

where, i and t represent the number of bank and time period. Y_{it} represent the total cost of bank i in time period t; LOA_{it} and OBS_{it} are the output variables (loan and the off-balance sheet items) of bank i in period t; POF_{it}, $POFA_{it}$ and POL_{it} is the price of input variables (price of fund, price of fixed assets and the price of labour) of bank i in period t; v is a two-sided error term, u is a non-negative technical inefficiency component and β's are unknown parameters. The parameters for each variable is obtained by banks from (1), the technical efficiency level for SOCBs is estimated by (2).

$$TE_{it} = \frac{Y_i}{e^{\beta_0+\beta_1 LOA_{it}+\beta_2 OBS_{it}+\beta_3 POF_{it}+\beta_4 POFA_{it}+\beta_5 POL_{it}+v_{it}+u_{it}}} = e^{-uit} \quad (2)$$

The empirical cost inefficiency effects model can be written as:

$$u_{it} = \delta_1 NII_{it} + \delta_2 NPL_{it} + \delta_3 ROA_{it} + \delta_4 ROE_{it} + \delta_5 CAR_{it} + \omega_{it} \quad (3)$$

where u_{it} is the inefficiency component; (NII_{it} is the non-interest income, NPL_{it} is the non-performing loan, ROA_{it} is the return on assets, ROE_{it} is the return on equity, CAR_{it} is the capital adequacy ratio of bank; ω_{it} is the error term.

2.2.2 Empirical Cobb-Douglas Stochastic Frontier Profit Model

The specification of Cobb-Douglas stochastic frontier profit model (Battese & Coelli, 1995) is defined as:

$$ln(\pi_{it} + \theta) = \beta_0 + \beta_1 LOA_{it} + \beta_2 OBS_{it} + \beta_3 POF_{it} + \beta_4 POFA_{it} + \beta_5 POL_{it} + v_{it} + u_{it} \quad (4)$$

where $\pi_{it}$is the profit after tax; θ is a constant. All the independent variables are the same line as the described in the equation (1). The technical efficiency is estimated as like as the equation (2) and the profit inefficiency effects model can be estimated as the same line as the equation (3).

2.2.3 Empirical Stochastic Translog Cost Frontier Model

The empirical specification of Stochastic Translog Cost frontier model is defined by

$$\begin{aligned}
&ln(C_{it}) = \beta_0 + \beta_1\, ln(LOA_{it}) + \beta_2\, ln(OBS_{it}) + \beta_3\, ln(POF_{it}) + \beta_4\, ln(PFA_{it}) + \\
&\beta_5\, ln(POL_{it}) + \frac{1}{2}[\beta_{11}\, ln(LOA^2{}_{it}) + \beta_{22}\, ln(OBS^2{}_{it}) + \beta_{33}\, ln(POF^2{}_{it}) + \beta_{44}\, ln(PFA^2{}_{it}) \\
&+\beta_{55}\, ln(POL^2{}_{it})] + \beta_{12}\, ln(LOA_{it}) * ln(OBS_{it}) + \beta_{13}\, ln(LOA_{it}) * ln(POF_{it}) \\
&+\beta_{23}\, ln(OBS_{it}) * ln(POF_{it}) + \beta_{45}\, ln(PFA_{it}) * ln(POL_{it}) + \beta_{14}\, ln(LOA_{it}) * ln(PFA_{it}) \\
&+\beta_{15}\, ln(LOA_{it}) * ln(POL_{it}) + +\beta_{24}\, ln(OBS_{it}) * ln(PFA_{it}) + \beta_{25}\, ln(OBS_{it}) * ln(POL_{it}) \\
&+\beta_{34}\, ln(POF_{it}) * ln(PFA_{it}) + \beta_{35}\, ln(POF_{it}) * ln(POL_{it}) + V_{it} + U_{it} \qquad (5)
\end{aligned}$$

where, C_{it} is the total cost. All the independent variables are the same line as the described in the equation (1). The empirical cost inefficiency effects model can be estimated as the same line as the equation (3) with the exception on the total cost in Translog case.

2.2.4 Empirical Stochastic Translog Profit Frontier Model

The specification form of stochastic Translog profit frontier model is defined as:

$$\begin{aligned}
&ln(\pi_{it} + \theta) = \beta_0 + \beta_1\, ln(LOA_{it}) + \beta_2\, ln(OBS_{it}) + \beta_3\, ln(POF_{it}) + \beta_4\, ln(PFA_{it}) + \\
&\beta_5\, ln(POL_{it}) + \frac{1}{2}[\beta_{11}\, ln(LOA^2{}_{it}) + \beta_{22}\, ln(OBS^2{}_{it}) + \beta_{33}\, ln(POF^2{}_{it}) + \beta_{44}\, ln(PFA^2{}_{it}) \\
&+\beta_{55}\, ln(POL^2{}_{it})] + \beta_{12}\, ln(LOA_{it}) * ln(OBS_{it}) + \beta_{13}\, ln(LOA_{it}) * ln(POF_{it}) \\
&+\beta_{14}\, ln(LOA_{it}) * ln(PFA_{it}) + \beta_{15}\, ln(LOA_{it}) * ln(POL_{it})\beta_{23}\, ln(OBS_{it}) * ln(POF_{it}) + \\
&\beta_{24}\, ln(OBS_{it}) * ln(PFA_{it}) + \beta_{25}\, ln(OBS_{it}) * ln(POL_{it}) + \beta_{34}\, ln(POF_{it}) * ln(PFA_{it}) \\
&+\beta_{35}\, ln(POF_{it}) * ln(POL_{it}) + \beta_{45}\, ln(PFA_{it}) * ln(POL_{it}) + V_{it} - U_{it} \qquad (6)
\end{aligned}$$

Where π_{it} is the profit after tax; θ is a constant. All the independent variables are the same line as the described in the equation (1). The empirical profit inefficiency effects model can be estimated as the same line as the equation (3) with the exception on the profit after tax in Translog case.

2.2.5 Empirical Tobit Regression Model

The specification of the Tobit regression model can be defined as:

$$E_{it} \;=\; \varphi_0 + \varphi_1 ITE_{it} + \varphi_2 ITI_{it} + \varphi_3 ITIN_{it} + \varphi_4 ITP_{it} + \varphi_5 ITPE_{it} + \varphi_6 ATMT_{it} + \varphi_7 ATME_{it} + \varphi_8 CCT_{it} + \varphi_9 CCE_{it} + \xi_{it}. \tag{7}$$

where E_{it} is defined as the Stochastic Cobb-Douglas and Translog cost and profit efficiency scores of the i-th bank in period t; ITE_{it} is the IT expanse of bank i in period t, ITI_{it} is the IT income of bank i in period t, $ITIN_{it}$ is the IT investment of bank i in period t, ITPit is the IT personnel of bank i in period t, $ITPE_{it}$ is the IT personnel expenses of bank i in period t, $ATMT_{it}$ is the ATM transaction of bank i in period t, ATMEit is the ATM expenses of bank i in period t, CCT is the Credit Card Transaction of bank i in period t, CCE_{it} is the credit card expenses of bank i in period t for the IT determinants of bank. ξit is the error term.

2.2.6 Likelihood Ratio Tests

The likelihood ratio test helps us to determine whether Stochastic Cobb-Douglas or Translog model is better or not. It provides other likelihood ratio tests where the null hypotheses are that there is no technical inefficiency and there is no interaction effect on the Translog stochastic cost and profit models. It is measured as follows:

$$\lambda = -2\{ln[L(H_0)/L(H_1)]\} = -2\{ln[L(H_0)] - ln[L(H_1)]\}. \tag{8}$$

where $L(H_0)$and $L(H_1)$are the values of the likelihood functions under the null and alternative hypothesis (note that this statistic has a mixed chi-square distribution). The null hypothesis is rejected when$\lambda_{LR} > \chi_c{}^2$.

3. Results

3.1 Cost and Profit Efficiency Estimation Based on Cobb-Douglas and Translog Stochastic Frontier Models

The results of maximum likelihood estimates of SOCBs using stochastic Cobb-Douglas frontier cost and profit models are given in Table 2. In this study, we have not found any significant estimates in case of stochastic frontier Cobb-Douglas cost model. Among the input prices, the price of fixed assets β_4 (-0.179) & the Price of labor β_5 (-0.032) were negative for the cost model. On the other hand, all the variables have a positive impact on the profit model and the significant estimates was found for loan (0.815) in case of stochastic frontier Cobb-Douglas profit model. The results of maximum likelihood estimates of both Stochastic Translog cost and profit frontier models show that the output variables, loan β_1 (6.57) was highly significant. Among the inputs variable, the price of fund and price of labor β_3

(-13.3) and β_5 (-19.7) were significant and negative for the cost model. Also the square outputs, price of fund β_{33} (-0.377) and price of labor β_{55} (-3.24) were negatively significant and the mixed product of loan & price of fund β_{13} (1.11), loan & labor β_{15} (0.268), off-balance sheet items & price of fixed assets β_{24} (0.268), price of fixed assets & price of fund β_{46} (0.504) were found to be significant but positive for the cost model and only the loan & price of fixed assets β_{14} (-0.399) was negatively significant.

On the contrary, in Translog stochastic profit frontier model, the majority of the variables were significant for the profit of SOCBs. The output variable loan β_1 (-47.88) and the inputs prices, price of fixed assets β_4 (-31.92) and price of labor β_5 (-61.76) were negative for the profit model whereas the output variable off-balance sheet items β_2 (15.73) and the inputs price of fund β_3 (47.24) was positive with significant. Also, the square outputs, loan β_{11} (5.08) and the square input price of fund β_{33} (1.72) shown positively significant and also the square input fixed assets β_{44} (-4.789) and the price of labor β_{55} (-12.75) were negatively significant. The mixed product between loan & off-balance sheet items β_{12} (-1.77), loan & price of fund, β_{13} (-3.26), off-balance sheet items & price of fund β_{23} (-1.59), price of fund & price of fixed assets β_{34} (-5.24), Price of fixed assets & price of labor β_{45} (-1.02) had found negatively significant where loan & price of labor β_{15} (1.54), off-balance sheet items & price of fixed assets β_{24} (2.89) and price of fund & price of labor β_{35} (2.26) were positively significant.

3.2 Cost and Profit Inefficiency Effect Estimation from Cobb-Douglas and Translog Stochastic Frontier Models

The results of cost and profit inefficiency estimates of SOCBs for the stochastic Cobb-Douglas cost and profit frontier models are given in Table 3. Indeed, all the variables were insignificant from the inefficiency cost model but the non-performing loan ($\delta_{2)}$ and the capital adequacy ratio (δ_5) were negative (-0.253) and (-0.162). Note that a negative sign indicates a negative impact of the variable on the bank inefficiency and therefore a positive effect on cost efficiency. Also in profit inefficiency model, the coefficient of return on assets δ_3 (-0.555) was a negatively significant indication that the banks were more profitable on less investment.

Table 2. Results on both Cost and Profit Efficiencies of SOCBs with Cobb-Douglas and Translog Stochastic Frontier Models

		Coefficient of Cost		**Coefficient of Profit**	
Variable	Parameter	Cobb-Douglas	Translog	Cobb-Douglas	Translog
Constant	β_0	0.039	-72.3***	0.194	180.81***
LOA	β_1	0.765	6.57***	0.815***	-47.88***
OBS	β_2	0.079	1.07	0.066	15.73***
POF	β_3	0.122	-13.3***	0.174	47.24***
POFA	β_4	-0.179	-1.01	0.151	-31.92***
POL	β_5	-0.032	-19.7***	0.029	-61.76***
(LOA)2	β_{11}		0.040		5.08***
LOA * OBS	β_{12}		-0.258		-1.77***
LOA * POF	β_{13}		1.11***		-3.26***
LOA * POFA	β_{14}		-0.399*		-0.432
Loan *POL	β_{15}		1.24***		1.54***
(OBS)2	β_{22}		0.197		0.421
OBS * POF	β_{23}		-0.263		-1.59**
OBS *POFA	β_{24}		-0.268***		0.462
OBS*POL	β_{25}		0.155		2.89***
(POF)2	β_{33}		-0.377*		1.72***
POF * POFA	β_{34}		-0.504*		-5.24***
POF *POL	β_{35}		-0.241		2.26***
(POFA)2	β_{44}		0.326		-4.789***
POFA *POL	β_{45}		0.131		-1.02**
(POL)2	β_{55}		-3.24***		-12.75***

*, ** . *** means significant at10%, 5% and1% level

Table 3. Cost and Profit Inefficiency Estimates of SOCBs with Cobb-Douglas and Translog Stochastic Frontier Models

		Coefficient of cost		Coefficient of Profit	
Variable	Parameter	Cobb-Douglas	Translog	Cobb-Douglas	Translog
NII	δ_1	0.371	0.896***	-0.161	-0.142
NPL	δ_2	-0.253	-0.681***	0.263**	0.228
ROA	δ_3	-0.059	-0.166***	-0.555***	-0.277
ROE	δ_4	0.012	0.0667	0.048	-0.129
CAR	δ_5	-0.162	-0.645***	-0.131	-0.566
Sigma Sq	σ^2	0.353	0.0202***	0.112**	0.065*
Gamma	γ	0.967	0.100***	1.00***	0.918**

*, ** . *** means significant at10%, 5% and1% level

As can be seen that the coefficient of non-performing loan δ_2 (0.263) was negatively significant implies that the bank could not be able to maximize profit if they have much non-performing loan. Besides the capital adequacy ratio was insignificant but negative with the coefficient of δ_5 (-0.131). The Sigma squared was found positive and significant. The estimated gamma γ (1.00) shows that the strong impact of inefficiency score to bank’s profit variance. The cost and profit inefficiency estimates for SOCBs show that the majority of the estimates were significant except return on equity. The coefficient of non-interest income δ_1 (0.896) was highly significant for the cost inefficiency model suggests that non-interest income did not impact banks cost. Also non-performing loan δ_2 (-0.681), return on assets δ_4 (2.38) and capital adequacy ratio δ_5 (-0.645) were negatively significant indication that they could be importantly influenced on banks profit efficiency. The Sigma squared was positively significant. The estimated gamma was close to unity implies that the strong impact of inefficiency score to bank’s cost variance. In profit inefficiency model, the coefficient of non-interest income δ_1 (-0.142), return on assets δ_3 (-0.277), return on equity δ_4 (-0.129) and capital adequacy ratio δ_5 (-0.556) were negatively insignificant. So, therefore, a positive effect on profit efficiency. The Sigma squared was positive and significant. The estimated gamma is 0.918 implies that the strong impact of inefficiency score to bank’s profit variance.

3.3 Results of Generalized Likelihood-Ratio Tests for both Stochastic Cost and Profit Frontier Models of SOCBs

The results of hypothesis tests were obtained for both cost and profit frontier models using the generalized likelihood-ratio statistic shown in Table 4.

The 1st null hypothesis is $H_0\colon \rho = 0$which specifies that the Cobb-Douglas stochastic frontier model is more preferable than the Translog stochastic frontier model for both cost and profit efficiency models of SOCBs. From this result, it was observed that the null hypothesis is rejected in both cases of cost and profit efficiency models. So the Translog model was more preferable than the Cobb-Douglas for cost and profit efficiency models of SOCBs.

Table 4. Generalized Likelihood-Ratio Test Results of both Stochastic Cost and Profit Frontier Models of SOCBs

Model	Null Hypothesis	Log-Likelihood Function	Test Statistics λ	Critical Value	Decision
Cost	$H_0\colon \rho_i = 0$		-54.96	38.301	Reject H_0
	Cobb-Douglas	-65.61			
	Translog	-38.13			
SOCBs	$H_0\colon \gamma = 0$	32.30	-0.0043	35.82	Accept H_0
	$H_0\colon \beta_{ij} = 0$	-7.37	18.377	5.138	Reject H_0

Profit	$H_0: \rho_i = 0$		16.31	35.83	Reject H_0
	Cobb-Douglas	-165.32			
	Translog	-157.17			
SOCBs	$H_0: \gamma = 0$	34.03	2.799	35.82	Accept H_0
	$H_0: \beta_{ij} = 0$	-7.94	36.98	5.14	Reject H_0

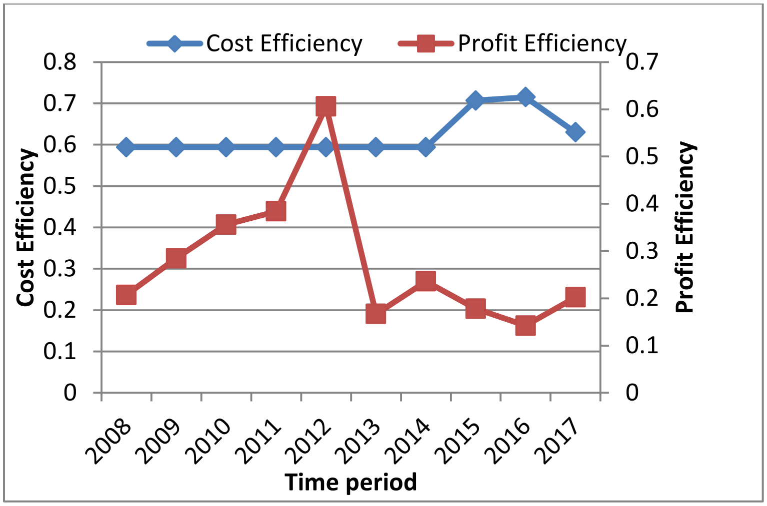
Note: all critical values are at 5% level of significance and the critical values are obtained from table of Kodde and Palm (1986).

The 2nd null hypothesis is$H_0: \gamma = 0$, which specify that there is no technical inefficiency effect in the cost efficiency model. The hypothesis is accepted for the state-owned commercial bank so there is a technical inefficiency effect in the cost model. In terms of profit model, the hypothesis is accepted for SOCBs, so we can conclude that there is a no technical inefficiency effect in the profit model for SOCBs.

The 3rd null hypothesis is $H_0: \beta_{ij} = 0$ which specifies that there is an interaction effects on both Translog stochastic cost and profit frontier models. From the result, it is observed that the null hypothesis is rejected in term of both cases of both cost and profit efficiency. So we can conclude that there is an interaction effect in the Translog stochastic cost and profit frontier models for SOCBs. This result was supported by (Thi and Ngan, 2014).

3.4 Year-wise Average Cost and Profit Efficiency Estimation of SOCBs using Cobb-Douglas Stochastic Frontier Model

The average profit and cost efficiency scores for the sample of SOCBs are illustrated in Figure 1. As can be seen that the average profit efficiency was 27.6% less than the average cost efficiency of 59.2%. SOCBs was fluctuated year by year in both cases of cost and profit. The profit efficiency of SOCBs was recorded 20% to 40% in the year 2008-2010, and then it attained the peak percentage amount of 60.6% in 2012. After that, it fell dramatically by 16.7% in 2013 which was the lowest percentage over the study period. Next year it has been slightly increased by 6%. Finally, it was observed that they again attained a slow upward trend of around 17% to 20% until 2017. The cost efficiencies were around 53% to 60% from 2008-2010, then it declined dramatically at the percentage of 38.4% of the year 2015 after then it increased over the period and the highest cost efficiency level was 71.5% in 2016.



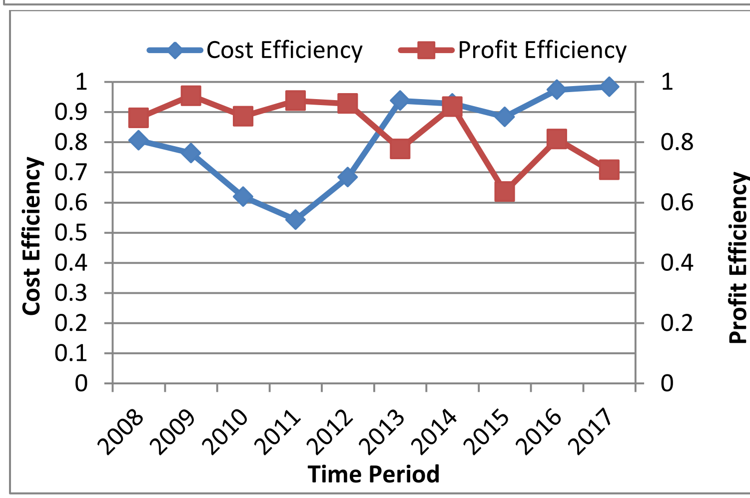


Figure 1. Year-wise Efficiency using Cobb-Douglas Model
Figure 2. Year-wise Efficiency using Translog Model

3.5 Year-wise Average Cost and Profit Efficiency of SOCBs using Translog Stochastic Frontier Model

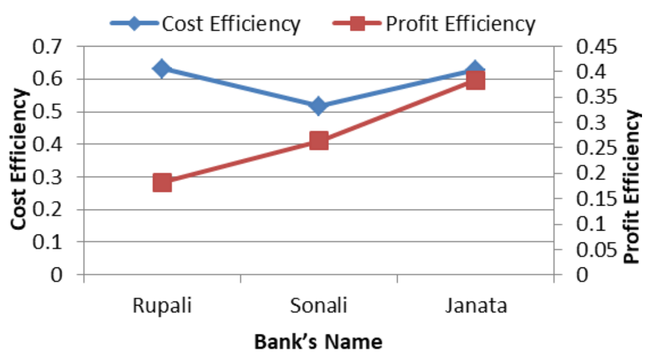
The year-wise average cost and profit efficiency scores for the sample of SOCBs are illustrated in Figure 2. It is observed that the average cost efficiency of 81.3% was less than the average profit efficiency of 84.4%. SOCBs have fluctuated year by year in both cases of cost and profit. The profit efficiency of SOCBs was recorded 80% to 90% from 2008 to 2014, and then it fell dramatically at 63.7% in2015. After then again it went up at 80% in 2016 and last year of the study period it declined by 10%. Conversely, at the beginning of the study period, the cost efficiency was 80.6% then it decreased slightly in the next two year at 76.4% to 68.4% respectively after then it rose just over 90% from 2013-2014, and again it has been slightly decreased by 88.4% in 2015. Finally, it has been an upward trend for the last three years and it attained the peak percentage amount of 98.4% in 2017.

3.6 Bank-wise Average Cost and Profit Efficiency of SOCBs using Cobb-Douglas Stochastic Frontier Model

The average cost and profit efficiency scores of individual SOCBs from 2008-2017 are reported in Figure 3. The Janata bank was the most cost & profit efficient among the other banks with an average efficiency score of 62.8% and 38.4% respectively. Besides Sonali bank was the second cost efficient (51.7%) and profit efficient (26.4%) and Rupali bank was the less cost and profit efficient with the score of 63.2% and 18.2% respectively.

3.7 Bank-wise Average Cost and Profit Efficiency of SOCBs for Translog Stochastic Frontier Model

The average cost and profit efficiency scores of individual SOCBs from 2008-2017 are shown in Figure 4. The Janata bank was the most efficient among the other banks with an average cost and profit efficiency score of 90% and 89.5% respectively. Besides Rupali bank was the second cost inefficient (89%) and profit efficient (84%) and Sonali bank was the less cost and profit efficient with the score of 63.8% and 79.7% respectively.



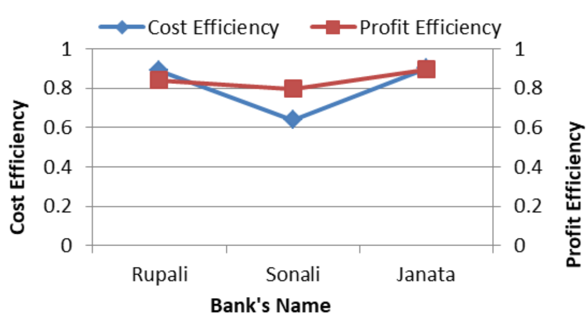


Figure 3. Bank-wise Efficiency using Cobb-Douglas Model
Figure 4. Bank-wise Efficiency using Translog Model

3.8 Results on Information Technology Determinants of both Cost and Profit Efficiency with Stochastic Cobb-Douglas and Translog frontier Models for SOCBs by using Tobit Regression

Table 5 represents the results of IT determinants of cost and profit efficiency of both stochastic Cobb-Douglas and Translog frontier models for SOCBs. In case of Cobb-Douglas model, it can be observed that the IT personnel expanses ϕ_5 (-0.002) was negatively significant for the cost efficiency of the SOCBs. On the other hand, In IT determinant profit efficiency of Stochastic Cobb-Douglas profit frontier model, the IT personnel expanses ϕ_5 (0.0015) was positively significant and IT investment ϕ_3 (-0.00009) and credit card expanses ϕ_9 (-0.0003) had a negative impact on the profit efficiency of SOCBs. In case of Translog frontier model, it can be seen that there was no significant IT variable for the cost efficiency but The IT expenses ϕ_1 (-0.0003), IT personnel expanses ϕ_5 (-0.002), ATM expenses ϕ_7 (-0.008) and credit card transaction ϕ_8 (-0.0004) had a negative impact on the cost efficiency of SOCBs. Also, there were not found significant IT variable for the profit efficiency but The IT income ϕ_2 (-0.0008), IT investment ϕ_3 (-0.00009), IT personnel ϕ_4 (-0.002), ATM transaction ϕ_6 (-0.0003) had a negative impact on the profit efficiency of SOCBs.

Table 5. Information Technology Determinants of Cost and Profit Efficiency Scores by Tobit Regression Model

		Coefficient of Cost		Coefficient of Profit	
Variable	Parameter	Cobb-Douglas	Translog	Cobb-Douglas	Translog
Intercept	ϕ_0	0.555***	0.623***	0.38***	0.948***
IT. Expenses	ϕ_1	-0.00002	-0.0003	-0.0001	0.0008
IT. Income	ϕ_2	-0.0001	0.002	0.00004	-0.0008
IT. Investment	ϕ_3	0.0001	0.0002	-0.00009***	-0.00009
IT. personnel	ϕ_4	0.002	0.003	0.0008	-0.002*
IT personnel expanses	ϕ_5	-0.002**	-0.002***	0.0015***	0.0006
ATM Transaction	ϕ_6	0.0002	0.00003	0.00004	-0.0003
ATM expeses	ϕ_7	-0.012	-0.008	-0.0002	0.004
Credit. Card. Transaction	ϕ_8	-0.0003	-0.0004	0.000001	0.0003
Credit. Card. expenses	ϕ_9	0.005	0.005	-0.0003***	0.008

*, ** . *** means significant at10%, 5% and1% level

4. Discussion

Year wise and bank-wise cost and profit efficiency of SOCBs have been measured using both Cobb-Douglas and Translog stochastic frontier cost, and profit functions proposed by Battese and Coelli (1995) for State-owned Commercial Banks. The input variables such as loan, off-balance sheet, price of fund, price of fixed asset, and price of labor contributed positively to increase the profit efficiency of SOCBs with Cobb-Douglas model while price of fixed asset, and price of labor played a role to decrease the cost of SOCBs. These results are found in line with the works of both (Altunbas et al., 2000 & Abdul-Majidet al., 2011). Both non-performing loan and capital adequacy ratio variables of SOCBs decreased the cost inefficiency while non-interest income, return on assets and capital adequacy ratio variables of SOCBs contributed to increase the profit efficiency with Tobit regression model. These results were supported by Raoudha (2010).

The SOCBs variables of price of fund, price of labor, mixed of loan and off-balance sheet items, mixed of loan and price of fixed asset, mixed of off-balance sheet items and price of fund, mixed of price of fund and price of fixed asset played a role to decrease the cost efficiency while off-balance sheet items, price of fund, mixed of loan and price of labor, mixed of off-balance sheet items and price of labor, mixed of price of fund and price of labor increased the profit efficiency of SOCBs with Translog model. These results were supported by (Huang, 2000 and Hasan and Marton, 2003). The variables of non-performing loan, return on assets and capital adequacy ratio decreased the cost efficiency while the variables such as non interest income, return on assets, return of equity, and capital adequacy ratio increased the profit efficiency of SOCBs by using Tobit regression model. These results were in line of the study of both (Raoudha, 2010 and Thi & Ngan, 2014).

Based on Cobb-Douglas model estimation results, the SOCBs used only 59.2% of their cost on an average during the study period to produce the same level of potential output while they earned only 27.6% potential profits where SOCBs have possibility to recover 72.4% profitability with the combination of available bank inputs where necessary. These result were not supported by (Raoudha, 2015 and Thi & Ngan, 2014) who measured the cost and profit efficiency of Vietnam commercial banks over the years from 2007 to 2012. Their results provided the profit efficiency around 61-68% and the cost efficiency about from 8% to nearly 20% over the sample period. In addition, SOCBs was found more efficient than other domestic commercial banks and foreign banks in case of profit efficiency.

Among SOCBs, Janata bank was found relatively most cost and most profit efficient with an average efficiency score of 62.8% and 38.4% respectively. These implies that Janata bank had costs 37.2%, were wasted relative to the best practice banks producing the same output and Janata bank earned only 38.4% potential profits where it has possibility to recover 61.6% profitability with the combination of available bank inputs where necessary or without increasing the bank outputs. These results confirmed that SOCBs were more cost efficient rather than profit efficiency. Translog stochastic frontiers showed that the average levels of cost and profit efficiency were found around 84% and they were quite stable over time. On an average SOCBs were wasted 19.7% relatively to the best practice banks as the average cost efficiency of SOCBs (81.3%) has been incurred only during the sample period while the average profit efficiency of 84.4% has been produced relative to the best performing banks. These results were supported by (Aiello & Bonanno, 2013) who evaluated both cost and profit efficiency of Italian banking sector over 2006-2011. The average bank-wise cost and profit efficiency were found 81.3% and 84.4% respectively that implied that these SOCBs were able to 18.7% more cost efficient and 15.6% more profit efficient by choosing optimum quantities and input prices. These results were in line with the study of (Thi & Ngan, 2014).

5. Limitation of the Study

Specifically the study undertook to investigate the use and development of some classes of IT applications which are as follows: ATM, Credit Cards, IT personnel expenses and IT investment from the selected SOCBs but some other IT factors cannot be gathered from the banks as they are not willing to disclose data due to their competitive reasons.

6. Conclusion

This study developed an appropriate stochastic frontier model in measuring both cost and profit efficiency of SOCBs and evaluated the impact of ICT component on both cost and profit efficiency using Tobit regression model. Translog model was found more preferable than Cobb-Douglas for both cost and profit efficiency. By using Cobb-Douglas frontier model, the average cost efficiency was found 59.2% whereas the average profit efficiency was 27.6% only. The highest cost efficiency occurred in 2016 while the highest profit efficiency in 2012. Janata bank was recorded the most cost and most profit efficient and Rupali bank was the less cost and less profit efficient. In case of Translog frontier model, the average cost efficiency was observed 81.3% and the average profit efficiency was 84.4%. Janata bank was observed the most cost and most profit efficient bank while Sonali bank was the less cost and profit efficient. IT personnel expanses was found negative and significant with Cobb-Douglas cost frontier model while IT personnel expanses was recorded positive and significant in Cobb-Douglas profit frontier model. IT expenses, IT personnel expanses, ATM expenses and credit card transaction had negative impacts on the cost efficiency of SOCBs while IT income, IT investment, IT personnel, ATM transaction had negative impacts on the profit efficiency of SOCBs in Translog frontier model. The results obtained from this study can help government, regulators, and investors to remove the hindrance of progress in economy of Bangladesh.

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Analysis of Rainfall Frequency Trend in Pre-Monsoon and Monsoon Season over Bangladesh

Md. Bazlur Rashid*
Meteorologist, Bangladesh Meteorological Department, Dhaka

Khadija Khatun
Department of Applied Mathematics, University of Dhaka

Afruza Sultana
Assistant Meteorologist, Bangladesh Meteorological Department, Dhaka

A.K.M. Nazmul Hoque
Meteorologist, Bangladesh Meteorological Department, Dhaka

Abstract

This study has been conducted on the basis of climate data such as rainfall of 30 selected stations for the period of 37 years from 1981 to 2017. Data are collected from Bangladesh Meteorological Department in daily basis. In this paper, the changing pattern of rainfall frequency in Bangladesh has been investigated. For this study, the Mann-Kendall test have been used to run 5% significance level on time series data for each station over the study period to detect the trend in rainfall frequency. We have observed that, rainfall frequency in pre-monsoon season are decreasing over all but not significantly. Here, it can easily understand that very heavy rainfall in March and moderately heavy to very heavy rainfall in April are decreasing significantly and the maximum frequency is 0.271/ year of very heavy rainfall in April. Their decreasing rates vary from 0.164 to 0.271. The light rainfall frequency in May of pre-monsoon and the whole monsoon season are increasing significantly. The increasing rates vary from 0.168 to 0.292 and the peak of increasing rate is 0.292/year, which occurs in July and the rest of the categories are insignificant. Spatial distribution of light and moderate to very rainfall frequency has been shafted in Bangladesh at decadal based, especially moderate to very rainfall frequency is changing rapidly in the country. From our study, it is projected that rainfall frequency in pre-monsoon season will slightly increase but in monsoon season it will decrease in the future.

***Keywords**:* Rainfall Frequency, Mann-Kendall Test, Trend Analysis, Spatial Distribution.

1. Introduction

Bangladesh is situated in the tropical monsoon county and its weather or climate is influenced by high temperature, heavy rainfall, often excessive humidity and is also depended on seasonal variations. The most prominent element of its climate is the reversal of the wind rotation between summer and winter, which is a fundamental part of the rotation system of the South Asian subcontinent. Bangladesh is a subtropical monsoon climate region. The climate of this country can be described under four seasons: Winter or Northeast Monsoon (December – February), which is characterized by very light northerly

* Corresponding author: bazlur.rashid76@gmail.com

winds, mild temperature and dry weather and clear to occasionally cloudy skies with fog over the country and the mean temperature is in the range of 18-20^{o}C; Summer or Pre-Monsoon (March-May), the average temperature during the summer season holds within 23-30^{o}C, April and May are the hottest months in this country and the highest maximum temperature ranging from 36-40^{o}C is attained in the north-western and south-western districts. Southwest Monsoon (June-September), in this season, the surface wind changes to south-westerly/southerly direction over the southern and the central districts and to south-easterly over the northern districts of the country, generally rain with widespread cloud cover and high humidity are the characteristics of this season, more than 71% of the total annual rainfall occurs in this season (Khatun et. al. 2016). The transition of season from the summer monsoon to the winter take places through Autumn or Post-Monsoon (October-November), when rainfall decreases noticeably. The dry period starts over the country and only 8% of the total annual rainfall occurs in this season.

Rainwater is very vital for the economic growth, disaster managing and hydrological planning for the country. An increasing in intensity of heavy rainfall events in Bangladesh through the simulation of variability and extremes of daily rainfall was predicted by May (2004). In the study of Rani et. al. (2014), where the rainfall data of Coimbatore for a period of 106 years (1907-2012) was analyzed, it is predicted that the rainy days are increasing but the changes were not statistically significant. An analysis of rainfall data for the generation of intensity duration frequency relationships showed that it has no significant change in rainfall intensities by Rimi et. al. (2016).

In this study, the changing pattern of rainfall frequency in Bangladesh has been investigated. The Mann-Kendall (MK) test is used to run 5% significance level on time series data for each station for the period of 1981 to 2017 and for future projection, Holt-Winters statistical model has been used here.

2. Data Sources

All categories of weather data like daily or monthly scale are existing at Bangladesh Meteorological Department (BMD), Agargaon (Dhaka), Bangladesh**.** In the present study, we have collected thirty-seven years' (1981-2017) daily rainfall data in millimeter(*mm*) of 30 rainfall stations throughout Bangladesh from BMD. To perform statistical analysis, monthly data have been prepared by taking mean of the daily data. Rainfall data from March to September have been classified as: Light rain (1-10 *mm*), Moderate rain (11-22 *mm*), Moderately heavy rain (23-44 *mm*), Heavy rain (45-88 *mm*), and very heavy rain (>88 *mm*).

3. Methodology

The approximation, extrapolation and calculation of trends and related statistical and numerical implication are important aspects of climate study. The rate at which rainfall changes over a time period is known as trend of the rainfall (Ahasan et. al. 2010). In the trend investigation, parametric and nonparametric equally are used broadly. Regression analysis methods are used for parametric purposes. Nonparametric approaches deal a smart alternative in this concern. Nonparametric prototypes for simulating rainfall fluctuate from the traditional procedures. The extremity performance of the data does not improperly impact the probability distribution in the core group of the data, and successive dependency is well-maintained in a more overall logic. As a consequence, the demonstration of separate outlier events may not be any better than that realized through the general parametric methods. But, the properties of classifications, including the statistics of a run of outlier events, may be better denoted. Removing the extremes data, nonparametric methods may provide an effective tool. The MK test can be used to measure monotonic trend (linear or non-linear) implication. So, in this study, we have used the MK test to detect the trend of rainfall frequency in Pre-monsoon as well as monsoon seasons in Bangladesh.

3.1 Mann- Kendall (MK) Test

The MK experiment developed by Mann, H. B. (1945) and Kendall, M. G. (1975) is generally used for tendencies classifying in climatologic and weather data in time series analysis. There are two benefits of using the MK test - (i) data need not to be normally distributed, and (ii) sensitivity reduces to be in a minimum level due to non-homogeneity of time series data (Tabari *et al*, 2011). The hypothesis for the test are as follows:

H_0: No monotonic trend.
H_1: Monotonic trend is present.

The MK test statistic (S) is given by:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sign(T_j - T_i).$$

$$Sign(T_j - T_i) = \begin{cases} 1 \; if \; T_j - T_i > 0 \\ 0 \; if \; T_j - T_i = 0 \\ -1 \; if \; T_j - T_i < 0 \end{cases}.$$

Where, T_j and T_i are the monthly values in months j and i, $j > i$ respectively.

If $n < 10$, the value of $|S\,|$ is shares straight to the theoretical distribution of S derived by MK. The two tailed experiment is used. At definite likelihood level H_0 is rejected in favor of H_1 if the original value of S equals or exceeds a certain value $S_{\alpha/2}$, where $S_{\alpha/2}$ is the lowest S which has the possibility less than $\frac{\alpha}{2}$ to execute in case of no trend. A positive (negative) value of S designates an upward (downward) trend. For $n \geq 10$, the statistic S is almost normally distributed with the mean and variance as follows:

$$E(S) = 0.$$

The σ^2 for the S statistic is expressed by:

$$\sigma^2 = \frac{n(n-1)(2n+5) - \sum t_i(i)(i-1)(2i+5)}{18}.$$

Where, t_i's represent the number of ties to extent i. The synopsis term in the numerator is used only if the data series contains tied values. The standard test statistic Z_s is considered as bellows:

$$Z_s = \begin{cases} \frac{S-1}{\sigma} \; for\; S > 0 \\ 0 \; for\; S = 0 \\ \frac{S+1}{\sigma} \; for\; S < 0 \end{cases}.$$

The test statistic Z_s is used a quantity of significance of trend. In detail, this experiment statistic is used to experiment the null hypothesis, H_0. If $|Z_s|$ is greater than $Z_{\alpha/2}$, where α denotes the special implication level (eg: 5% with $Z_{0.025} = 1.96$) then the null hypothesis is inacceptable suggesting that the trend is important.

3.2 Nearest Neighbor (NN) Method

Spatial patterns or spatial distribution of data are generated by NN method. NN procedures are among the most popular methods used in statistical pattern recognition (Holmes et. al. 2002). The models are conceptually simple and empirical, studies have shown that their performance is highly competitive against other techniques such as Kriging, Inverse distance a power, triangulation with linear interpolation etc. It turns out that whether or not the data are regular or irregular is unimportant once anybody define what is known as NN.

Let $\mathcal{R} = \{s_i : i = 1, 2, \dots n\}$ where, $\mathcal{R} \subset \mathcal{R}^d$, for d a positive integer, be the set of all sites making up a finite lattice. Two sites are said to be neighbors, if the response variables at these sites depend on each other directly. Let $N_i = \{S_k\, ;\, S_k \text{ is a neighbor of } s_i\}$, $i = 1, 2, \dots, n$. Then the set $G_{\mathcal{R}} = \{S_i, N_i : 1, 2, \dots, n\}$ is defined as a NN for $\mathcal{R}$, with the following two properties:

1. $s_i \notin N_i, i = 1, 2, \dots, n$ (a site is not a neighbor of itself).
2. $s_j \in N_j \Rightarrow s_j \in N_i,\ \forall_i, j = 1, 2, \dots, n$.

Example of NN: Suppose a realization $y = \{y\,(s_i): i = 1,2, \dots, n\}$ is taken from a regular $n_1 \times n_2$ grid of sites in $\mathcal{Z}^2$ (the two dimensional integer space) where, $n = n_1 \times n_2$ is the number of sites in the lattice. For this setup, a two dimensional coordinate (j, k) corresponds naturally to each site S_i. Hence, this two dimensional lattice $\mathcal{R}$ and corresponding realization can be defined by:

$$\mathcal{R} = \{(j,k); j = 1,2,\ldots,n_1 \ and \ k = 1,2,\ldots n_2\}.$$

$$y = \{y_{jk}; j = 1,2,\ldots,n_1 \ and \ k = 1,2,\ldots n_2\}.$$

The first-order neighborhood system consists with the horizontal and vertical adjacent units. Precisely, this neighborhood system is defined with the sites for which the following criteria is satisfied.

$$N(j,k) = \left\{(j^{'},k^{'}) \colon (j-j^{'})^2 + (k-k^{'})^2 = 1\right\}.$$

The second-order neighborhood system for a given site (i,j) is defined the sites for which the following criteria is satisfied

$$N(j,k) = \left\{(j^{'},k^{'}) \colon (j-j^{'})^2 + (k-k^{'})^2 \leq 2\right\}.$$

The same idea can be extended for defining a neighborhood system of higher order. Lattice or areal data units are neighbors if they are within a distance d. Distance based neighboring suggests that, one can create distance bins such as $(0,d_1], (d_1,d_2], (d_2,d_3],$ and so on, to extend the concept of neighborhood system in terms of first-order neighborhood, second-order neighborhood, third-order neighborhood and so on, respectively. In this type of neighborhood, those units who are within $(0,d_1]$ distance from unit i are said to be first order neighbor of unit i, those units who are within $(d_1, \ d_2]$ distance from unit i are said to be second order neighbor of unit i and so on.

3.3 Holt-Winters Exponential Smoothing Additive Model

Holt (1957) and Winters (1960) methods contain the forecast equation and three smoothing equations for- level, trend and seasonality with smoothing parameters. In this study, we have used Holt-Winters trend method to detect the trend in monthly rainfall frequency data of pre-monsoon and monsoon season. The component form for the additive model is

Forecast equation: $\hat{y}_{t+h|t} = l_t + hb_t$.
Level equation: $l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1})$.
Trend equation: $b_t = \beta * (l_t - l_{t-1}) + (1 - \beta *)b_{t-1}$.

Where, l_t denotes an estimate of the level of the series at time t, b_t denotes an estimate of the trend (slope) of the series at time t, α is the smoothing parameter for the level, $0 \leq \alpha \leq 1$, and $\beta *$ is the smoothing parameter for the trend, $0 \leq \beta * \leq 1$.

The level equation shows l_t is a weighted average of the observation y_t and the one step ahead training forecast for time t is given by l_{t-1}+b_{t-1}. The trend equation shows that, b_t is a weighted average of the estimated trend at time t based on $l_t - l_{t-1}$ and b_{t-1}, and h is the linear function of the forecast.

4. Results and Discussions

4.1 Trend of Rainfall Frequency in Pre-Monsoon Season

From Table 4.1, we observed that rainfall frequency in the pre-monsoon season are decreasing over all but not significantly. Here, it is clear that very heavy rainfall in March is decreasing at the rate 0.164/ year (Fig. 4.1.1), whereas light rain in May is increasing at the rate 0.257/year with 5% level of significance. In April, on the other hand, moderately heavy, heavy and very heavy rainfall is significantly decreasing, shown in Figs. (4.1.2 – 4.1.4)**,** where the highest decrement rate is 0.271 per year for very heavy rainfall.

Table 4.1. Trend of Rainfall Frequency in Pre-monsoon Season

Categories	March		April		May	
	τ	p	τ	p	τ	p
Light rain (1-10 *mm*)	-0.036	0.382	0.000	0.5	**0.257**	**0.013****
Moderate rain (11-22 *mm*)	0.017	0.448	-0.103	0.19	0.079	0.252
Moderately heavy rain (23-44 *mm*)e	-0.125	0.144	-0.225	**0.026***	0.101	0.194
Heavy rain (45-88 *mm*)**5-88)**	-0.053	0. 333	-0.224	**0.029***	-0.045	0. 352
Very heavy rain (>88 *mm*) **e**	**-0.164**	**0.099**$^{+}$	-0.271	**0.013***	-0.031	0.402

'+' indicates the presence of monotonic trend at 10% level and '*' indicates the presence of monotonic trend at 5% level of significance

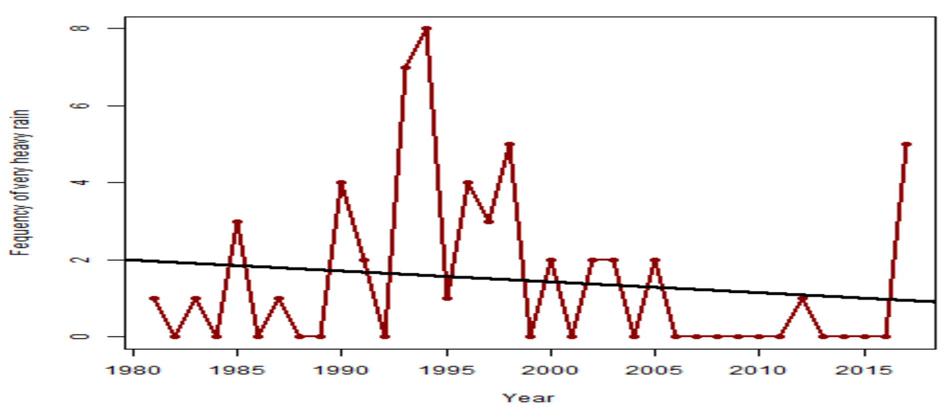

Figure 4.1.1. Trend Analysis of Frequency of Very Heavy Rain in March

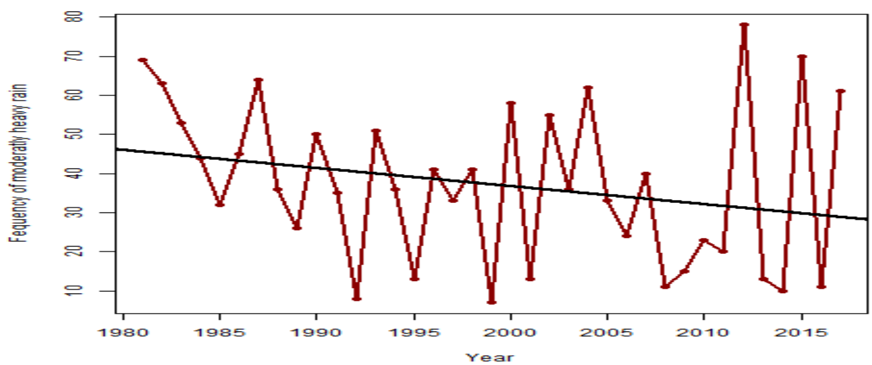

Figure 4.1.2. Trend Analysis of Frequency of Moderately Heavy Rain in April

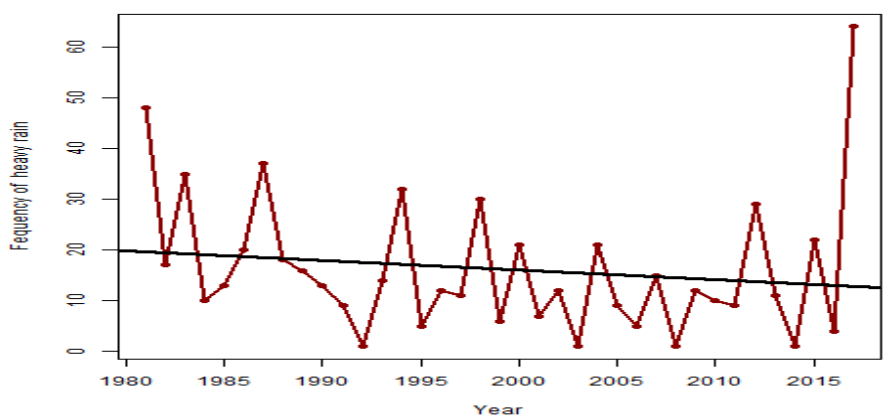

Figure 4.1.3. Trend Analysis of Frequency of Heavy Rain in April

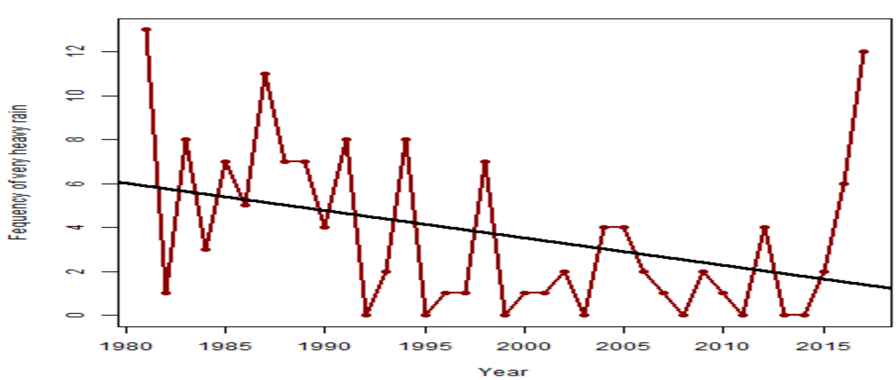

Figure 4.1.4. Trend analysis of frequency of very heavy rain in April

4.2 Trend of Rainfall Frequency in Monsoon Season

From this study, it is obvious that light rainfall frequency in the whole monsoon season are increasing significantly. In Table 4.2, we see light rainfall frequency are increasing with rates 0.168/year, 0.292/year, 0.246/year and 0.206/year in June, July, August and September respectively. The trend of these months are shown in Fig.4.2. We also observe that the highest increment rate is in July and it varies from 0.168 to 0.292. But, the trend of the rest of the categories are insignificant.

Table 4.2. Trend of Rainfall Frequency in Monsoon Season

Categories	June		July		August		September	
	τ	p	τ	p	τ	p	τ	p
Light rain (1-10 *mm*)	0.168	0.07^{+}	**0.292**	**0.006***	0.246	**0.017***	0.206	**0.038***
Moderate rain (11-22 *mm*)	-0.064	0.295	-0.002	0.5	0.094	0.212	-0.018	0.443
Moderately heavy rain (23-44 *mm*)e	0.059	0.309	0.076	0.269	**0.265**	**0.011***	0.019	0.219
Heavy rain (45-88 *mm*) **5-88)**	0.103	0.190	-0.006	0.484	0.116	0.159	0.052	0. 333
Very heavy rain (>88 *mm*) **e**	0.0272	0.412	0.138	0.119	0.22	0.427	-0.051	0. 338

'+' indicates the presence of monotonic trend at 10% level and '*' indicates the presence of monotonic trend at 5% level of significance

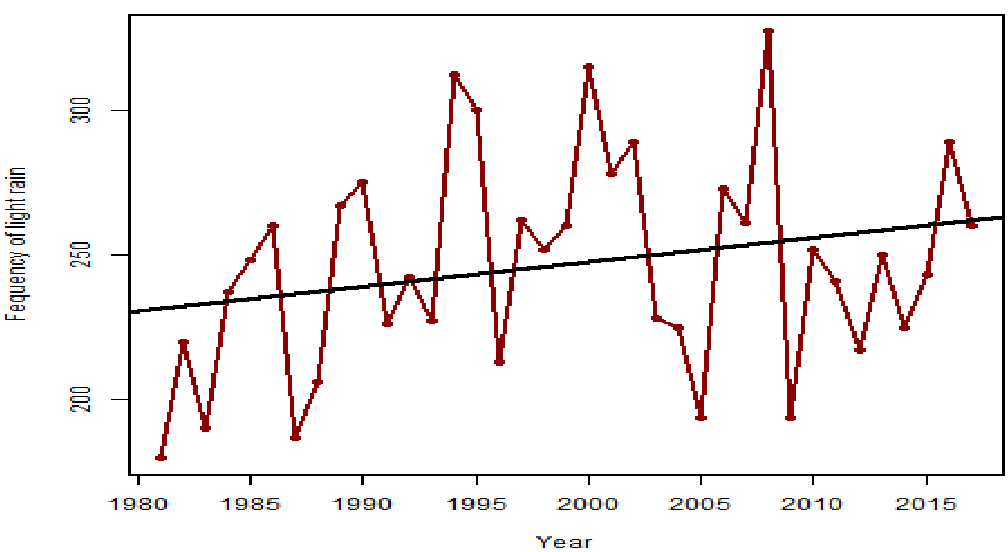

Figure 4.2.1. Trend Analysis of Frequency of Light Rain in June

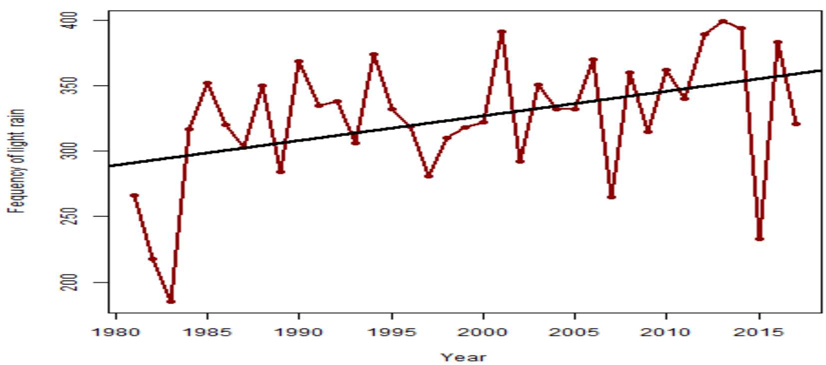

Figure 4.2.2. Trend Analysis of Frequency of Light Rain in July

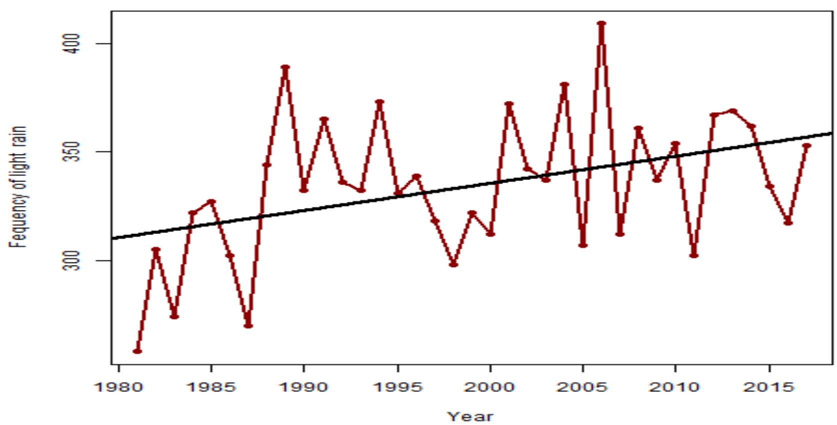

Figure 4.2.3. Trend Analysis of Frequency of Light Rain in August

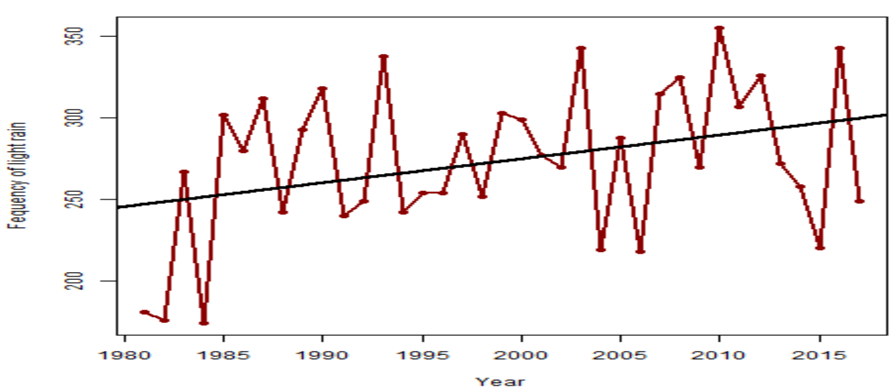

Figure 4.2.4. Trend Analysis of Frequency of Light Rain in September

4.3 Spatial distribution of decadal rainfall frequency in pre-monsoon season during (1981-2020)

Light rainfall frequency in the pre-monsoon season lies among 110-140 in the most part except the northeast and the south-southeastern part of Bangladesh during the period 1981-1990 (Fig. 4.3.1). But in 1991-2000, these areas become declined and it lies at the major central part and hilly regions of Bangladesh. After that 2001-2010 period, light rainfall frequency (140-170) has been increased in the north and the northeastern part of the country and it is continued in the next decade and in the last two decades, frequency (80-140) is also shifted and spread in some areas (Fig. 4.3.2 -4.3.4).

Moderate to very heavy rainfall frequency (80-140) in pre-monsoon season has captured in the major part of the country during 1981-1990 and it has increased at 100-200 in some areas in 1991-2000. But in the next decade 2001-2010, it has decreased in the southwestern part of the country. At the last decade 2011-2020, this frequency has again increased in the major part of the country (Fig. 4.3.5-4.3.8).

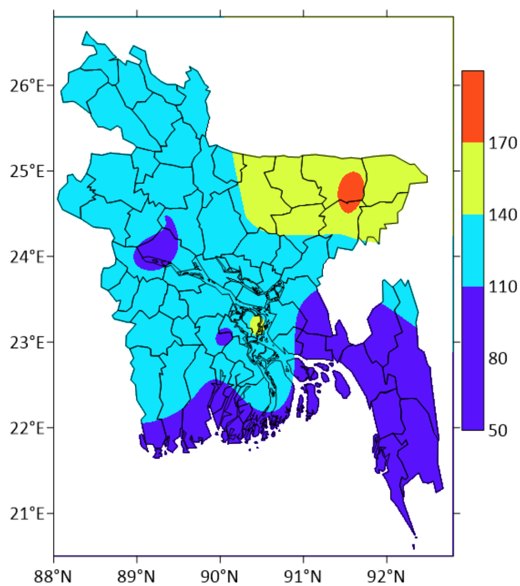

Figure 4.3.1. Light Rainfall in Pre-Monsoon Season During (1981-1990)

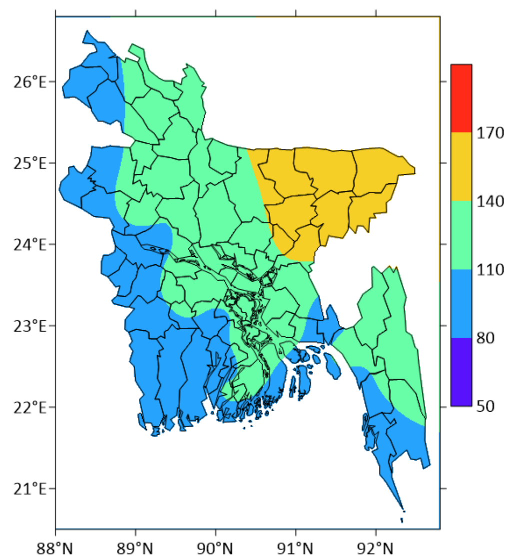

Figure 4.3.2. Light Rainfall in Pre-Monsoon Season During (1991-2000)

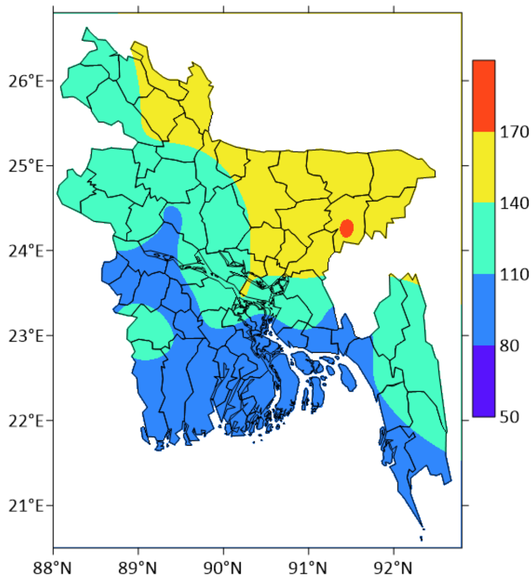

Figure 4.3.3. Light Rainfall in Pre-Monsoon Season During (2001-2010)

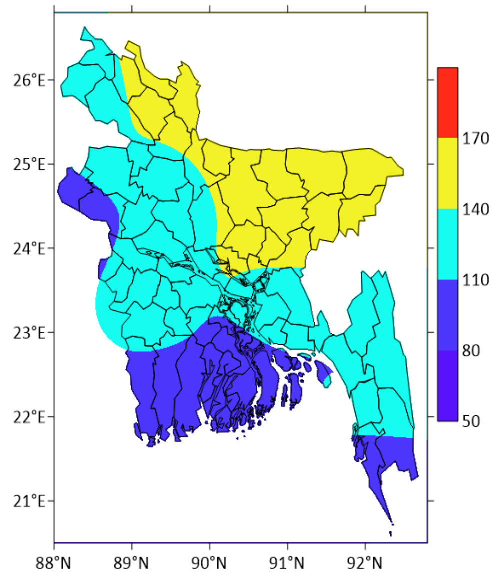

Figure 4.3.4. Light Rainfall Pre-Monsoon Season During (2011-2020)

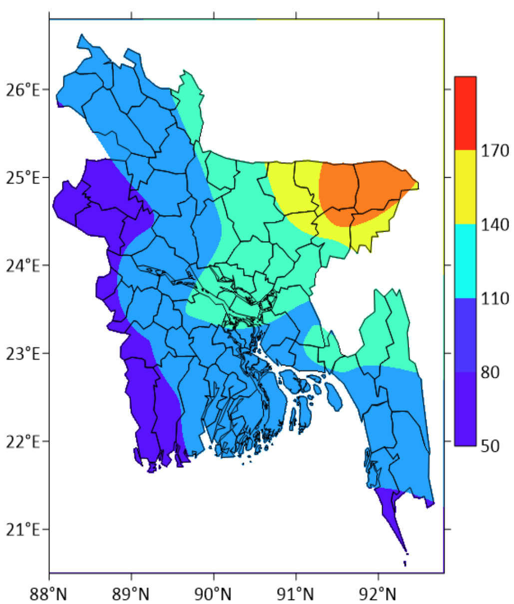

Figure 4.3.5. Moderate-Very Heavy Rainfall in Pre-Monsoon Season During (1981-1990)

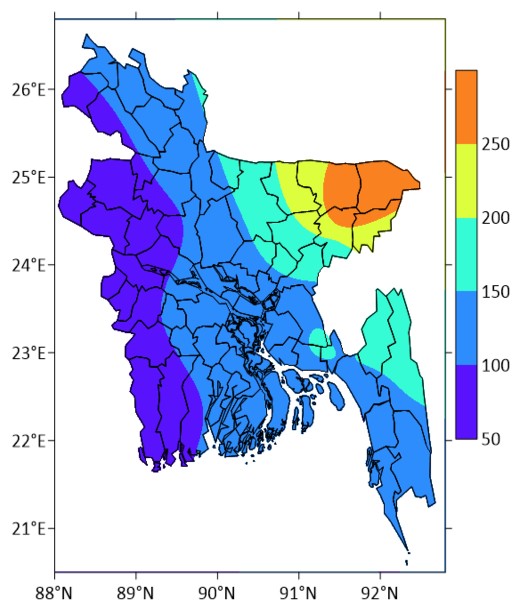

Figure 4.3.6. Moderate -Very Havy Rainfall in Pre-Monsoon Season During (1991-2000)

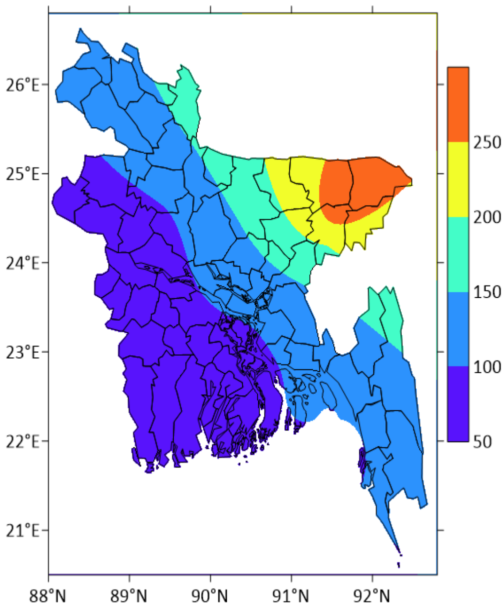

Figure 4.3.7. Moderate-Very Heavy Rainfall in Pre-Monsoon Season During (2001-2010)

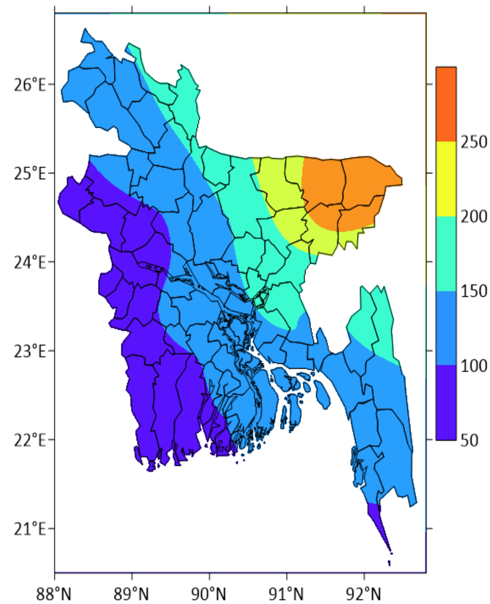

Figure 4.3.8. Moderate-Very Heavy Rainfall in Pre-Monsoon Season During (2011-2020)

4.4 Spatial distribution of decadal rainfall frequency in monsoon season during (1981-2020)

The highest light rainfall frequency in monsoon season 400 or more lies among the central part to the southwest part of Bangladesh in the decade 1981-1990. But, in the next decade (1991-2000) it has increased in the major part of the country except for the northwest and some part of Sylhet and Noakhali and it is remaining unchanged in the following decade (2001-2010). However, in the last decade (2011-2020) it is again going to the downward trend (Fig 4.4.1 - 4.4.4).

The number of moderate to very heavy rainfall frequency in monsoon season lies among 200 to 400 in the major part except for the northeast and the southeastern part of the country in the period 1981-1990. Almost the same conditions stay in the following decade (1991-2000). On the other hands, in some areas like Rajshahi, Pabna and Kushtia, the number of this frequency are decreasing in 2001-2010 and it is also rapidly decreased over the western part of Bangladesh in the last decade 2011-2020 (Fig 4.4.5 - 4.4.8).

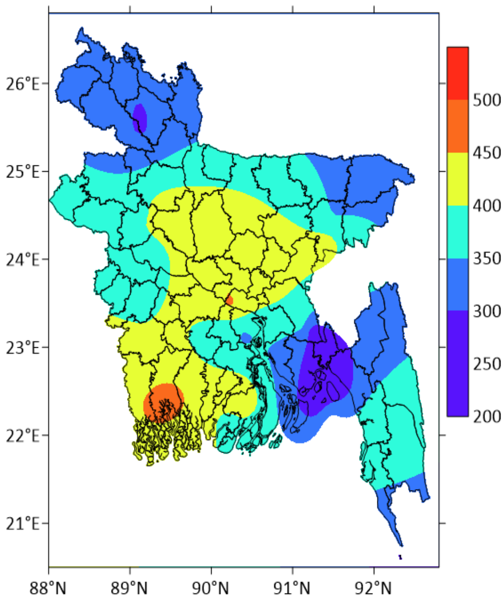

Figure 4.4.1. Light Rainfall in Monsoon Season During (1981-1990)

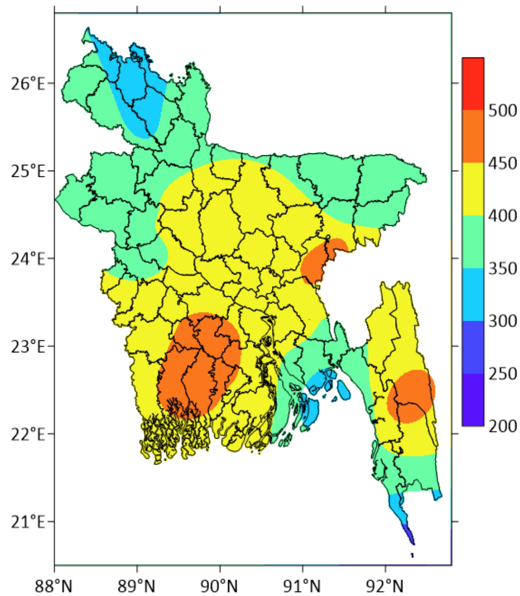

Figure 4.4.2. Light Rainfall in Monsoon Season During (1991-2000)

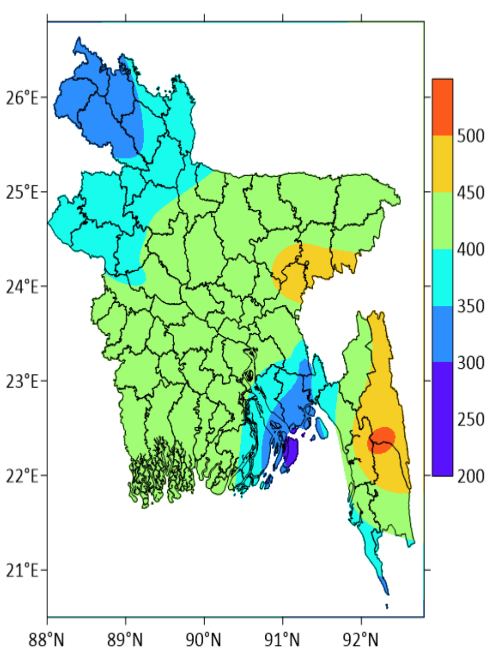

Figure 4.4.3. Light Rainfall in Monsoon Season During (2001-2010)

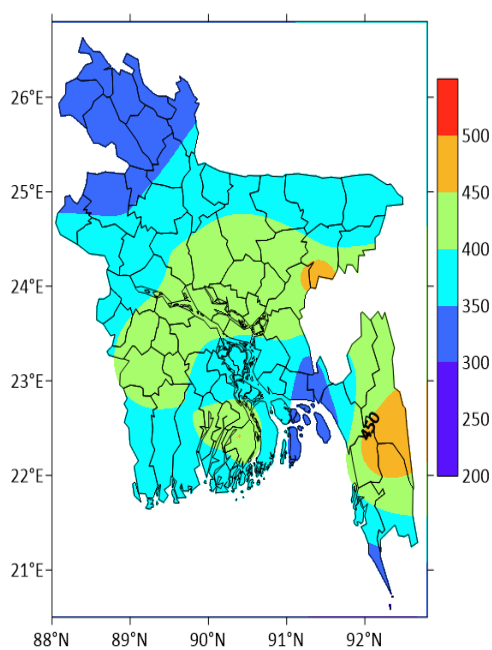

Figure 4.4.4. Light Rainfall in Monsoon Season During (2011-2020)

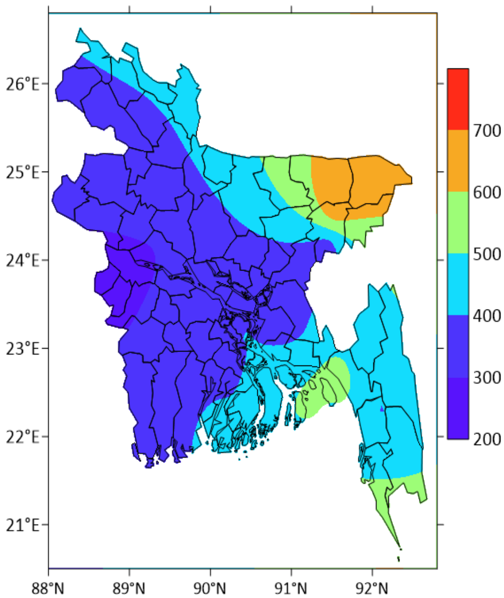

Figure 4.4.5. Moderate-Very Heavy Rainfall in Monsoon Season During (1981-1990)

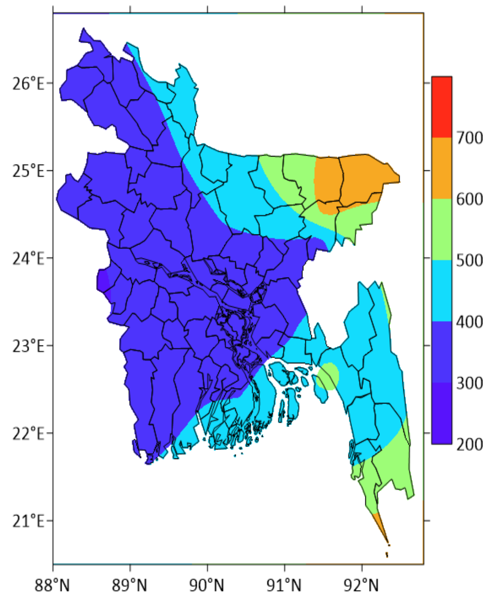

Figure 4.4.6. Moderate-Very Heavy Rainfall in Monsoon Season During (1991-2000)

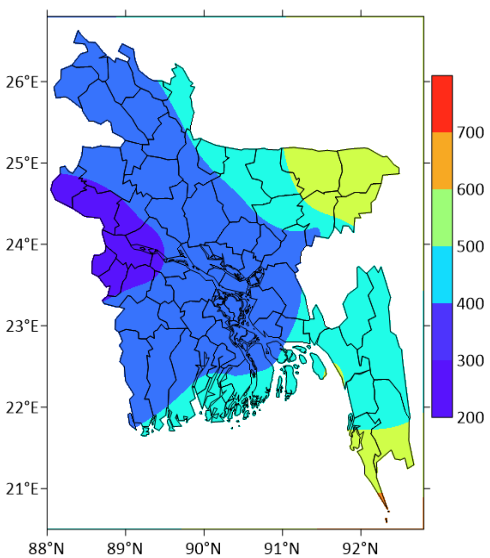

Figure 4.4.7. Moderate-Very Heavy Rainfall in Monsoon Season During (2001-2010)

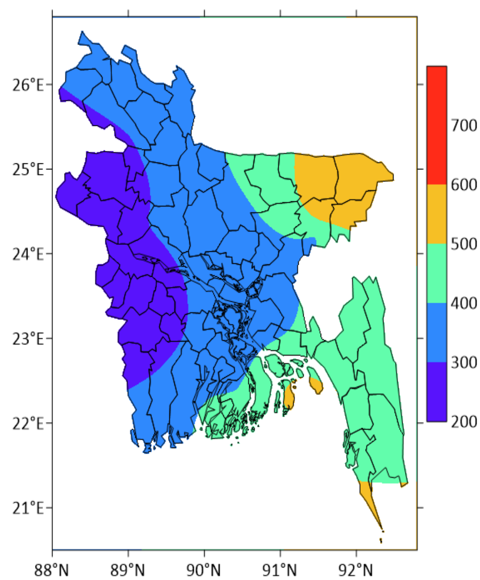

Figure 4.4.8. Moderate-Very Heavy Rainfall in Monsoon Season During (2011-2020)

4.5 Rainfall frequency forecast using Holt-Winters exponential smoothing method for pre-monsoon season during (2021-2030)

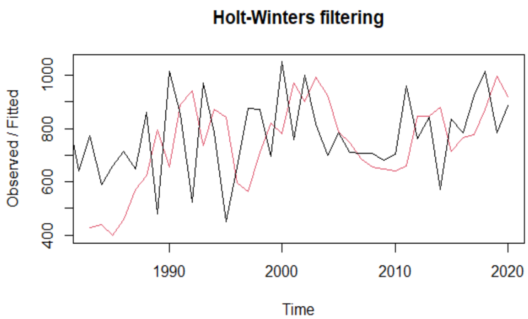

Figure 4.5.1. Holt-Winters Exponential Smoothing Applied to Pre-Monsoon Rainfall Frequency of Bangladesh During 1981-2020.

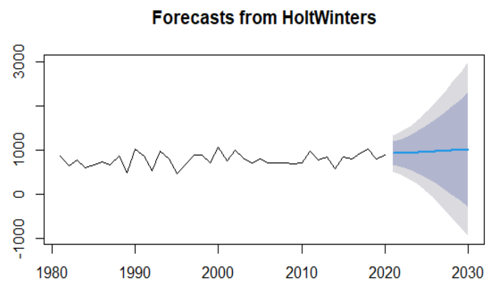

Figure 4.5.2. Forecast of Rainfall Frequency of Pre-Monsoon Season During (2021-2030) using Holt-Winters Exponential Smoothing Additive Model in Bangladesh.

In Fig. 4.5.1, the black line is the observed line and the red line is the model fitted line. For pre-monsoon season, the observed line of rainfall frequency and the model fitted line of rainfall frequency are closed, despite some years. So, the Holt-Winters Exponential Smoothing additive models' forecasts are good fitted with the observations. In Fig. 4.5.2, the blue line shows the forecasts with the 80% prediction interval as a deep shaded area and the 95% prediction interval as a light shaded area.

Table 4.3. Forecast for Rainfall Frequency in Pre-Monsoon Season During 2021-2030

Year	Forecast	Lowest (80%)	Highest (80%)	Lowest (95%)	Highest (95%)
2021	914.4249	648.24078	1180.609	507.3314	1312.518
2022	924.4676	606.5189	1242.416	438.2069	1410.728
2023	934.5102	542.2531	1326.767	334.6047	1534.416
2024	944.5529	459.3598	1429.746	202.5140	1686.592
2025	954.5955	361.3273	1547.864	47.2701	1861.921
2026	964.6382	250.6595	1678.617	-127.2979	2056.574
2027	974.6809	129.0786	1820.283	-318.5562	2267.918
2028	984.7235	-2.20901	1971.656	-524.6595	2494.107
2029	994.7662	-142.3261	2131.859	-744.6646	2733.799
2030	1004.8088	-290.6093	2300.227	-976.3623	2985.980

The estimated value of alpha is 0.4434968 and beta is 0.4729901 which indicate that, the level and the slope of the time series both change quite a lot over time. The sum of squared errors is 1626243. The forecasted rainfall frequency for the pre-monsoon season from 2021-2030 with 80% and 95% prediction intervals (lowest, highest) are shown in Table 4.3.

4.6 Rainfall frequency forecast using Holt-Winters exponential smoothing method for monsoon season during (2021--2030)

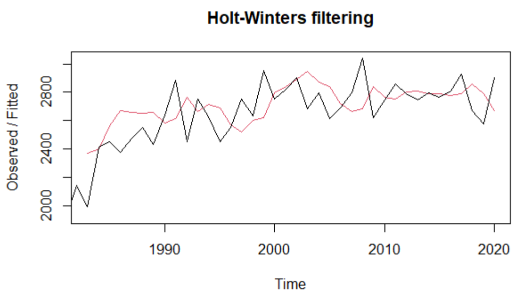

Figure 4.6.1. Holt-Winters Exponential Smoothing Applied to Monsoon Rainfall Frequency of Bangladesh During 1981-2020

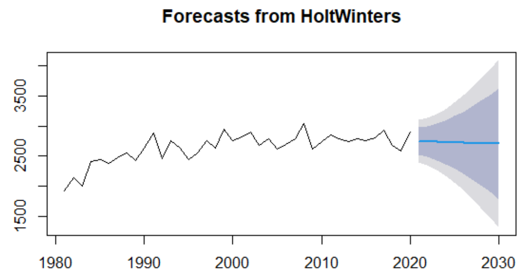

Figure 4.6.2. Forecast of Rainfall Frequency for Monsoon Season During (2021-2030) from Holt-Winters Exponential Smoothing Method in Bangladesh

In Fig. 4.6.1, the black line is the observed line and the red line is the model fitted line. For pre-monsoon season, the observed line of rainfall frequency and the model fitted line of rainfall frequency are mostly closed except some years. So, the Holt-Winters Exponential Smoothing additive models' forecasts are pretty good fitted with the observations. In Fig. 4.6.2, the blue line shows forecasts with the 80% prediction interval as a deep shaded area and the 95% prediction intervals as a light shaded area.

Table 4.4. Forecast for Rainfall Frequency in Monsoon Season During 2021-2030

Year	Forecast	Lowest (80%)	Highest (80%)	Lowest (95%)	Highest (95%)
2021	2746.263	2516.055	2976.471	2394.190	3098.336
2022	2741.568	2482.702	3000.434	2345.667	3137.469
2023	2736.873	2433.363	3040.384	2272.694	3201.053
2024	2732.178	2369.511	3094.846	2177.526	3286.830
2025	2727.484	2293.377	3161.591	2063.574	3391.393
2026	2722.789	2206.947	3238.631	1933.877	3511.701
2027	2718.094	2111.736	3324.452	1790.750	3645.439
2028	2713.399	2008.849	3417.949	1635.883	3790.915
2029	2708.705	1899.097	3518.312	1470.516	3946.893
2030	2704.010	1783.085	3624.935	1295.576	4112.443

The estimated value of the alpha and beta is 0.340226 and 0.511534, which indicate that the level and the slope of the time series both change quite a lot over time. The sum of squared errors is 1239350. The forecasted rainfall frequency for monsoon season from 2021-2030 with 80% and 95% prediction intervals (lowest, highest) are shown in Table 4.4.

5. Conclusion

In this analysis, all over the rainfall frequency in the pre-monsoon season are decreasing but not significantly. Here, it can easily understand that very heavy rainfall in March and moderately heavy to very heavy rainfall in April are decreasing significantly and the maximum frequency is 0.271/year in case of very heavy rainfall in April. Their decreasing rates vary from 0.164 to 0.271 per year. On the other hand, light rainfall frequency in May and the whole monsoon season are increasing significantly and the maximum increasing is in July and its increasing rates differ from 0.168 to 0.293 and the trend of the rest of the categories are insignificant. Spatial distribution of decadal rainfall frequency in pre-monsoon and monsoon season has changed in some particular areas of Bangladesh both light and moderate to very rainfall categories. Our study predicts that, rainfall frequency in pre-monsoon will slightly increase in future, however, in monsoon it will decrease in future gradually.

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Determinants of Mass Media Exposure and Involvement in Information and Communication Technology Skills among Bangladeshi Women

Md. Moyazzem Hossain*
Department of Statistics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

Arif Bin Saleh
Resident Physician, Jersey Shore University Hospital, New Jesrsey, USA

Mohammad Alamgir Kabir
Department of Statistics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

Abstract

Nowadays, it is clear that access to different media along with information and communication technology (ICT) enhances the overall development of a countries. Women empowerment plays a key role in poverty reduction as well as ensuring sustainable development. The purpose of this paper is to assess the status of media exposure i.e., radio, television, print/electronic media and ICT skills along with finding out the potential determinants of the media exposure and ICT skills by women aged 15-49 years in Bangladesh considering the MICS-2019 dataset. Among the three media, the findings reveal that television is very popular than newspaper and radio. Young women are more exposed to mass media, however, fewer ICT skills compared to adult women. The findings also disclose that age, education, marital status, living area, geographical location, overall happiness and wealth index are significantly linked with the status of mass media exposure as well as ICT skills of women in Bangladesh. Therefore, it is essential to take necessary steps to enhance the mass media exposure and increase the rate of ICT skills among women who lived in urban and rural areas to become self-confident as well as independent which will be helpful to accelerate the SDGs 4, 5 and 16 goals by ensuring 4.3, 4.4, 5.1, 5.b, and 16.10 targets in Bangladesh by 2030.

Keywords: Women reproductive age; Media exposure; Logistic regression; ICT communication skills.

1. Introduction

Bangladesh is trying to attain gender equality along with women empowerment to make sure the Sustainable Development Goals (SDGs) by 2030 (United Nations, 2015). Bangladesh is one of the heavily populated countries and its population is estimated 162.7 million, among them there are 81.3 million and 81.4 million women and men respectively in 2017 (Akhter Asma & Islam, 2019). In rural area, the percentage of women are higher than men, however, the scenario is different in urban area i.e., the ratio of women and men are almost same (Akhter Asma & Islam, 2019). The government established a national web gateway comprising 46,500 government offices and 5,875 Digital Centres as part of the a2i program to assure people's easy access to public services. Through the 4,571 Union Digital Centres, even individuals in remote areas can access a variety of services online. To meet the industry's human

* Corresponding author: hossainmm@juniv.edu

resource needs, the government has been working to generate educated experts in order to increase the number of IT professionals to 2 million by 2021. Furthermore, there are approximately 0.5 million freelancers that work on outsourcing projects (Bangladesh Planning Commission, 2020).

Nowadays, exposure to different print/electronic media plays a vital role to change norms, behaviours and habits human beings (Dasgupta, 2019). Media is helpful to create awareness regarding domestic violence as well as may be play a positive role to change the status of women like independent and empowered women, especially in low-literate backgrounds (Jesmin & Amin, 2017). Moreover, both political and job opportunities may also be circulated through mass media designed for women folk (Bhushan & Singh, 2014). Previous studies found that radio, television, and/or print media is potential to change gender norms associated to violent behaviour towards women (Bhushan & Singh, 2014; Bhattacharya, 2016; Jesmin & Amin, 2017). Ting, et al. (2014) show that exposure to different mass media strongly and positively influence women's regarding their rights and empowerment (Ting et al., 2014). Furthermore, throughout the previous few decades, campaigns available on different media have been utilized as an endeavour to improve numerous behaviours related to health among general populations (Robinson et al., 2014).

Along with mass media, the usage and execution of information and communication technologies (ICTs) have impacted to revolutionary economic as well as social transformation throughout the World (Unwin, 2009, 2010; Karim et al., 2011; Castells, 2013; Mukerji, 2013; Sey et al., 2013). However, Ullah (2017) claims that the empowerment due to ICTs does not achieved automatically following their complete implementation (Ullah, 2017). Unfortunately, women lived in rural areas have the less right and access to the technology (Rashid et al., 2010). However, to ensure the development of women, ICTs offers a wide-ranging opportunities which contribute positively for the gender equality (Afrah & Fabiha, 2017; United Nations Division for the Advancement of Women, 2005). Women are gradually more adopting ICT skills for numerous business associated tasks and enhance their knowledge of communication, marketing, purchasing etc. in different parts of the World (Ndubisi & Kahraman, 2005). Statistics reveal that individuals living in developing countries own up fewer computers, mobile phones and phone lines than developed countries (James, 2007). Women empowerment is one of the crucial basic elements of poverty reduction as well as ensuring sustainable development (Fox & Romero, 2017). Several previous studies conducted is several countries pointed out that, the utilization of ICT may be positive for the empowering women economically (Prasad & Sreedevi, 2007).

If Bangladeshi women were educated along with enabled to enrich their knowledge by various ICT tools, for example, mobile/smart phones, computers, and the internet then the level of poverty could be lessened and development/advancement would be achievable in economic, social, and every sectors of a person's life (Laizu et al., 2010). In Bangladesh, ICT can perform as a vital role in changing the lifestyle of women both in urban and rural regions through making them confident and self-independent (Laizu et al., 2010). Moreover, Ashraf el al., (2009) pointed out that the community participants accept the modern and advanced ICT knowledge and skills in the interior of their social as well as economic constraints (Ashraf et al., 2009). Considering the necessity, the present Government of Bangladesh are emphasizing on ICT sector and try to visualize Bangladesh as a "Digital Bangladesh by 2021" where information and communication technology is undoubtedly playing the magical role. However, Islam and Grönlund (2011) mention that there is an evidence of progressing slowly toward materializing a "Digital Bangladesh" (Islam & Grönlund, 2011).

Over the last couple of years there is a significant advancement observed in ICT sector of Bangladesh. To develop a digital society i.e., an ICT driven knowledge-based society, the present government is trying to facilitates people in different aspects. However, to make sure the SDSs it is necessary to involve women in the ICT sectors where they can be acting as a crucial role in family, community as well as social development. Moreover, it is essential to explore the factors that influence women to participate ICT activities. Therefore, the authors intended to assess the status of media exposure and ICT skills as well as to find the potential determinants of the media exposure and selected ICT activities by women aged 15-49 years in Bangladesh.

2. Methodology

2.1 Data

The data were taken from Multiple Indicator Cluster Survey-2019 (MICS-2019). Briefly, the 2019 MICS is considered a country wide and fully representative sample survey in Bangladesh conducted by BBS with the collaboration of UNICEF that provided information on different indicators of the eight administrative divisions of Bangladesh. The MICS utilized a two-stage stratified sampling procedure for collecting the required information. Firstly, the districts were taken as the primary sampling strata, as well as a number of census enumeration areas (EAs) were selected from every stratum. Then a systematic sample of 20 families was taken from every EA. This survey include 64,400 respondents. However, the missing observations were excluded from this study. Therefore, the subsequent analysis of this study is based on N=64,377 women in 2019 (Bangladesh Bureau of Statistics (BBS) & UNICEF Bangladesh, 2019).

2.2 Variables

The paper considers two target variables viz. media exposure and selected ICT skills. The first target variable is whether there is a media exposure (exposed to any media e.g., newspaper, radio and television at least once a week) or not and coded as "0" for no exposure and "1" for any exposure or Yes. To fulfil the objective of the research and simplicity of analysis, the answers to the questions on "frequency of listening to radio, reading newspaper and watching television" were categorized as follows: almost every day, at least once a week, less than once a week = 1 and not at all = 0. Hence, any respondent who chosen at least one 'yes' for all the three was considered as exposed to mass media and assign a value "1" and those who chosen 'no' for all the three questions were judged as no exposed to mass media and coded as "0".

However, the second outcome variable is the status of ICT skills i.e., involvement of selected ICT activities by women and is determined by completed at least one of the following nine itemized computer associated activities available in the MICS-2019 dataset and assigned 0 for "No" and 1 for "Yes" (Bangladesh Bureau of Statistics (BBS) & UNICEF Bangladesh, 2019).

> (i) "Copied or moved a file or folder"; (ii) "Used a copy and paste tool to duplicate or move information within a document"; (iii) "Sent e-mail with attached file, such as a document, picture or video"; (iv) "Used a basic arithmetic formula in a spreadsheet"; (v) "Connected and installed a new device, such as a modem, camera or printer"; (vi) "Found, downloaded, installed and configured software"; (vii) "Created an electronic presentation with presentation software, including text, images, sound, video or charts"; (viii) "Transferred a file between a computer and other device"; (ix) "Wrote a computer program in any programming language".

This study is intended to identify the association between the status expose to mass media, and status of ICT skills determined by several ICT activities by women with several socio-economic and demographic variables. The independent variables included in the analysis are: women reproductive age, geographical division (Barishal, Chattogram, Dhaka, Khulna, Mymensingh, Rajshahi, Rangpur, and Sylhet); educational status (pre-primary or none, primary, secondary and higher secondary+); currently married (yes, no); ethnicity of household head (Bengali, others (Chakma, Santal, Marma, Tripura, and Garo)); having functional difficulties (yes, no); place of living (urban, rural); overall happiness (very happy, somewhat happy, neither happy nor unhappy, somewhat unhappy, very unhappy); wealth index (poorest, poorer, middle, richer, and richest). The selection of variables used in this study was motivated by the availability in the MICS dataset and self-efficacy as well as guided by relevant literature(Ahinkorah et al., 2020; Dasgupta, 2019; Fatema & Lariscy, 2020; Rabbi, 2012; Ullah, 2017; Yaya et al., 2018).

Analysis

The percent distribution of different variables is calculated. Besides, the bivariate analysis (χ^2-test) is performed to determine the significant associations between the target variables and selected socio-demographic factors. Moreover, the association of the household ownership of different ICT equipment as well as accessibility of internet with several characteristics of the respondents were done. These associated variables were considered as the independent variables for the logistic regression model (adjusted), which was carried out in this study to find out the determinants of the status of media exposure and identify the most influential factors for involving ICT activities by women. The general form of a logistic regression model can be expressed as,

$$\Pr\left(Y_i = 1\right) = \frac{\exp\left(X_i\beta\right)}{1 + \exp\left(X_i\beta\right)}$$

where, Y_i takes a value of '1' if the respondent is having media exposure/participate to the selected ICT activities and '0' otherwise, X_i is a vector of covariates and β is a vector of parameters which contains the intercept parameter and the regression parameters associated with a set of covariates used in the study. The fitted form of the model can be defined as,

$$\ln\left[\frac{\hat{P}_i}{1-\hat{P}_i}\right] = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \ldots + \hat{\beta}_k X_k ,$$

where, $\hat{\beta}_l$ $(l = 0,1,2,\ldots,k)$ represents the estimated regression coefficient of the l^{th}independent variable in the study.

3. Results

The mass media acting as a vital sources of information as well as exposure to latest and new thoughts. The media also play as a significant role in a country where females have no or low education, fewer opportunities of employment outside the home, or employment on the family owned farm, or even limited autonomy of movement. Moreover, media exposure can be noticed as a cause of "empowerment" for women just like education. The percent distribution of exposed to mass media of 15-49 years old women are presented in **Table 1** along with different selected demographic and socio-economic characteristics. The results describe that the age and education of the participants are crucial for determining the status of media exposure to all three media individually as well as jointly. Among the three media, the findings reveal that television is very popular than newspaper and radio. There is an adverse association between age and rate of use media e.g., the highest number of women watch television (63.5%), listen to the radio (2.7%) of age 15-19 years and read a newspaper (5.6%) of age 20-24 years. Among the age group of women, 45-49 years of aged women have the lowest percentage of listening to the radio (0.47%) and watching television (55.7%). Higher educated women are more exposed to media. It is seen that more than one-fourth of women having education of higher secondary level or above use any media, however, less than half of the women whose educational status is none or pre-primary use any media among the three. Moreover, unmarried women and has no functional difficulties use more mass media than their counterpart [**Table 1**].

Table 1. The Percent Distribution of Exposed to Mass Media of 15-49 Years Old Women

Characteristics	One or more times in a week (%)				
	Read a newspaper (a)	Listen to the radio (b)	Watch television (c)	Any media (d)	All three media (e)
Age (in years) [a, b, c, d, e:*]**					
15-19	5.01	2.72	63.52	65.41	2.30
20-24	5.61	2.04	62.95	64.63	2.74
25-29	4.74	1.09	62.44	63.49	2.14
30-34	3.12	0.61	60.75	61.39	1.52
35-39	3.20	0.58	58.29	58.90	1.82
40-44	3.44	0.51	57.69	58.22	1.82
45-49	2.85	0.47	55.73	56.16	1.63
Education [a, b, c, d, e:*]**					
Pre-primary or none	0.1	0.16	43.33	43.52	0.15
Primary	0.3	0.39	52.06	52.38	0.14
Secondary	2.2	1.15	65.06	65.98	1.01
Higher secondary+	18.7	3.89	77.93	81.28	9.28
Currently married [a, b, c, d, e:*]**					
Yes	3.04	0.80	59.81	60.56	1.49
No	8.48	3.15	64.39	66.71	4.21
Has functional difficulty [a:, b:NS, c:***, d:***, e:NS]**					
Yes	3.15	1.24	49.18	50.03	1.75
No	4.09	1.08	60.62	61.60	2.03
Ethnicity of household head [a: NS, b: *, c:***, d:***, e:**]**					
Bengali	4.14	1.29	61.23	62.29	2.06
Others	3.91	0.40	40.19	40.98	1.33
Overall happiness [a, b, c, d, e:*]**					
Very happy	7.73	1.97	70.90	72.53	3.81
Somewhat happy	3.08	1.07	59.84	60.75	1.54
Neither happy nor unhappy	1.93	0.95	47.06	47.76	0.92
Somewhat unhappy	1.18	0.62	44.49	45.04	0.50
Very unhappy	1.94	0.28	43.15	43.80	0.65
Living area [a, b, c, d, e:*]**					
Urban	11.06	2.47	80.17	81.34	6.26
Rural	2.38	0.97	55.80	56.83	0.97
Division [a, b, c, d, e:*]**					
Barishal	2.67	1.80	37.25	38.91	1.22
Chattogram	4.82	0.81	55.92	57.09	2.11

Characteristics	One or more times in a week (%)				
	Read a newspaper (a)	**Listen to the radio (b)**	**Watch television (c)**	**Any media (d)**	**All three media (e)**
Dhaka	5.56	2.12	73.59	74.63	3.49
Khulna	4.12	1.17	67.16	68.15	1.88
Mymenshing	4.02	0.72	57.88	59.08	1.47
Rajshahi	3.11	1.57	69.19	69.85	1.81
Rangpur	3.10	0.77	54.94	55.77	1.19
Sylhet	3.63	0.49	49.76	50.97	1.38
Wealth Index [a, b, c, d, e:*]**					
Poorest	0.52	0.61	21.09	22.10	0.11
Poorer	1.05	0.74	51.39	52.29	0.36
Middle	1.87	0.85	67.07	68.01	0.69
Richer	3.78	1.51	78.74	79.87	1.53
Richest	14.96	2.87	89.29	90.66	8.39

Note: NS: Not significant; *, **, ***: Significant at 10%, 5% and 1% level respectively

The status of overall happiness is also highly significantly linked with the status of exposure to several mass media. The percentage of reading newspapers for the very happy women is about four times more than very unhappy women. The percentage of watching television by very happy women is approximately half of the very unhappy women. It is observed that women whore sided in rural regions are less exposed to mass media than women who currently lived in urban regions. For example, 11.1% percent of urban women read a newspaper but only 2.4% of rural women read a newspaper. About 60% and more than 80% of women use any media (newspaper, radio, or television) who lived in rural and urban areas respectively. The highest number of respondents who lived in the Dhaka division (74.6%) are more exposed to mass media (newspaper, radio or television) and the lowest number of women are lived in the Barishal division (38.9%). Moreover, women who are not financially solvent are less exposed to mass media. Only 22.1% of the poorest women use any media, however, just above 90% of women use any media whose wealth status is richest. This scenario is similar for all three media. The findings reveal that living area, geographical location and wealth status are significantly linked with the status of mass media exposure of women whose age lies between 15 to 49 years [**Table 1**].

The ownership of different ICT equipment like television, mobile, computer along with access to the internet according to several selected background characteristics is presented in **Figure 1**. It is seen that the percentage of the ownership of television, computer and internet access is almost equal for all age groups. The percentage of ownership of television, computer and access to the internet are about 50%, 5% and 35% respectively. However, the ownership of mobile phones varies among age categories, for example, the percentage of ownership of mobile phones is about 80% for 25-34 years of women and the lowest percentage is for age groups of 15-19 years. The results depict that more than 85% of women have a mobile phone with an education level higher secondary or above while 57.2% of illiterate or pre-primary educated women have a mobile phone. Women who lived in urban regions use more mobile phones than women who lived in rural parts of Bangladesh. The percentage of ownership of television is almost equal for all the categories of wealth index. A little bit higher percentage is observed in the ownership of computer and access to internet among the rich people compared to poor people. However, the rate of ownership of mobile phones is about 20% more among the richest women compared to poorer women **Figure 1**.

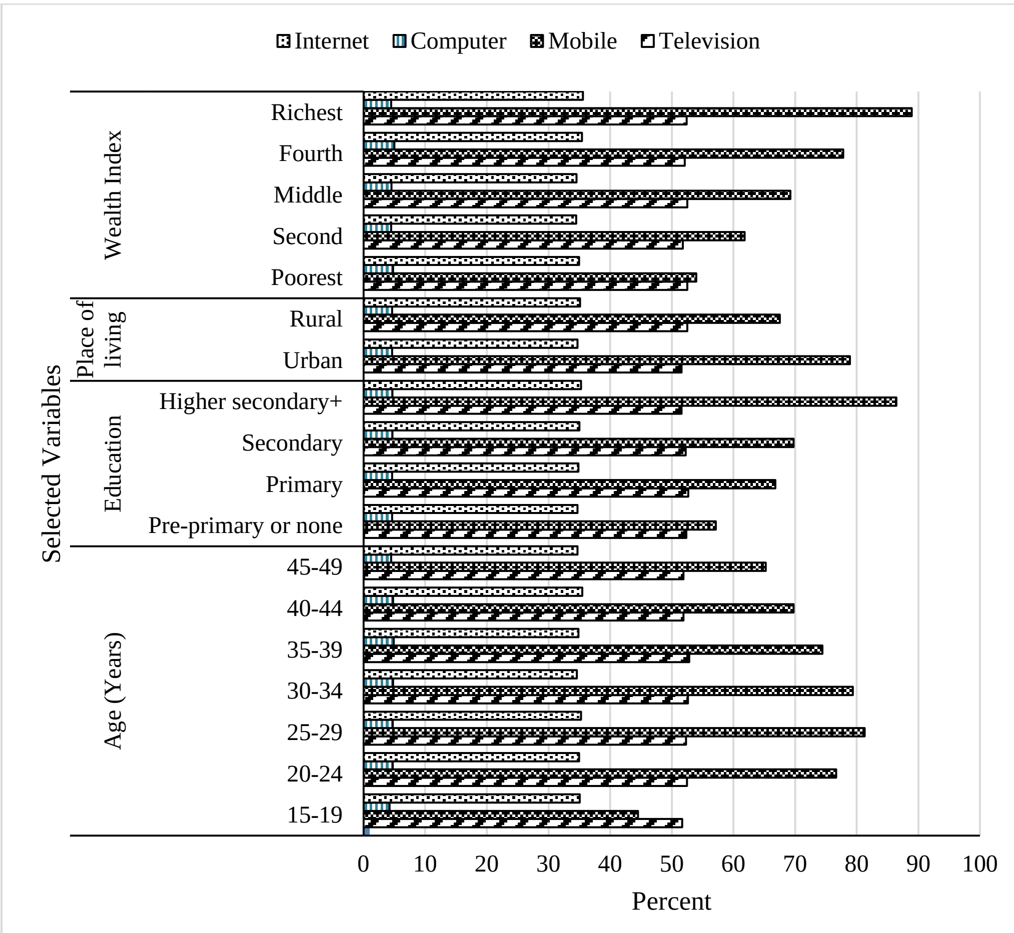

Figure 1. Ownership of ICT-Equipment along with Access to Internet by Selected Variables

Table 2. Results of Logistic Regression of the Status of Media Exposure and ICT Skills along with Selected Covariates

Characteristics	Labels	Media exposure		ICT skills	
		B	AOR (95% CI)	B	AOR (95% CI)
Age (in years)	15-19 (Ref.)				
	20-24	0.08	1.08 (1, 1.18)*	-0.23	0.8 (0.46, 1.4)
	25-29	0.16	1.17 (1.07, 1.27)***	-0.28	0.76 (0.41, 1.39)
	30-34	0.13	1.14 (1.05, 1.25)***	0.29	1.34 (0.63, 2.84)**
	35-39	0.04	1.04 (0.95, 1.13)	0.16	1.17 (0.51, 2.67)*
	40-44	0.02	1.02 (0.92, 1.12)	0.01	1.01 (0.43, 2.38)
	45-49	-0.11	0.89 (0.81, 0.99)**	0.85	2.33 (0.7, 7.73)**
Education	None or primary (Ref.)				
	Secondary	0.31	1.36 (1.28, 1.45)***	1.56	4.75 (0.47, 8.17)*
	Higher secondary+	0.60	1.82 (1.67, 1.98)***	2.14	8.52 (2.38, 15.18)*
Currently married	Yes (Ref.)				
	No	0.20	1.22 (1.14, 1.31)***	0.63	1.88 (1.25, 2.83)***
Functional difficulties	Yes (Ref.)				
	No	0.17	1.19 (1.06, 1.33)***	-0.07	0.93 (0.17, 5.12)
Ethnicity of household head	Bengali (Ref.)				
	Other	0.56	1.76 (1.53, 2.02)***	-0.28	0.75 (0.2, 2.84)
Overall happiness	Very happy (Ref.)				
	Somewhat happy	-0.02	0.98 (0.93, 1.03)	-0.27	0.76 (0.54, 1.09)**
	Neither happy nor unhappy	-0.30	0.74 (0.69, 0.79)***	0.11	1.12 (0.39, 3.2)
	Somewhat unhappy	-0.38	0.68 (0.6, 0.77)***	-0.11	0.9 (0.06, 12.49)*
	Very unhappy	-0.36	0.7 (0.6, 0.82)***	-0.92	0.4 (0.05, 3.47)**
Place of living	Urban (Ref.)				
	Rural	-0.48	0.62 (0.58, 0.65)***	-0.09	0.92 (0.62, 1.35)*
Division	Barishal (Ref.)				
	Chattogram	0.17	1.19 (1.1, 1.29)***	0.72	2.06 (0.88, 4.83)*
	Dhaka	0.96	2.62 (2.41, 2.85)***	0.95	2.59 (1.16, 5.78)**
	Khulna	0.89	2.44 (2.25, 2.65)***	0.97	2.62 (1.11, 6.2)**
	Mymenshing	0.94	2.56 (2.3, 2.85)***	0.48	1.61 (0.58, 4.44)
	Rajshahi	1.16	3.19 (2.91, 3.48)***	0.17	1.18 (0.5, 2.78)*
	Rangpur	0.80	2.23 (2.04, 2.43)***	1.74	5.7 (1.97, 16.53)***
	Sylhet	-0.04	0.96 (0.87, 1.06)	0.38	1.46 (0.53, 4.01)*
Wealth index	Poorest (Ref.)				
	Poorer	1.25	3.5 (3.29, 3.71)***	0.92	2.51 (0.24, 26.52)
	Middle	1.91	6.76 (6.35, 7.19)***	1.04	2.83 (0.05, 2.36)*

Characteristics	Labels	Media exposure		ICT skills	
		B	**AOR (95% CI)**	**B**	**AOR (95% CI)**
	Richer	2.16	8.67 (7.92, 9.54)***	0.82	2.27 (0.07, 2.79)**
	Richest	2.25	9.44 (8.42, 13.14)***	0.55	1.74 (0.09, 3.6)**
Constant		-1.87		-2.12	

Note: AOR: Adjusted odds ratio; CI: Confidence interval; *, **, *** represents significant at 10%, 5% and 1% level respectively

The results of logistic regression of media exposure status and status of ICT skills are presented in **Table 2**. It is observed that age, education, marital status, status of functional difficulties, overall happiness, geographical location of residence and wealth index are highly significant factors for the status of media exposure of women. The women of age 25-29 years have 17% (AOR: 1.17, 95% CI: 1.07-1.27) higher chance to expose to mass media compared to 15-19 years aged women, however, about 11% (AOR: 0.89, 95% CI: 0.81-0.99) less chance of using mass media by women of aged 45-49 years compared to 15-19 years aged women. The likelihood of media exposure is 1.82 times more (AOR: 1.82, 95% CI: 1.67-1.98) of women having higher secondary and above education compared to illiterate or primary educated women. Moreover, very unhappy women and lived in rural are less exposed than very happy women who resided in urban areas. With region, the highest likelihood of mass media exposure is observed in the Rajshahi division (AOR: 3.19, 95% CI: 2.91-3.48) and less exposed in the Sylhet division (AOR: 0.96, 95% CI: 0.87-1.06) as compared with the Barishal division. The richer (AOR: 8.67, 95% CI: 7.92-9.54) and richest (AOR: 9.44, 95% CI: 8.42-13.14) women have more than 8- and 9-fold higher odds for media exposure compared to the poorest women.

On the other hand, young women having fewer ICT skills than adult women. For example, women aged 25-29 years (AOR: 0.76, 95% CI: 0.41-1.39) have less odds compared to 15-19 years aged women but women aged 45-49 years (AOR: 2.33, 95% CI: 0.7-7.73) having 2.33 times higher likelihood of have ICT skills compared to age the group 15-19 years. Higher educated (e.g., Secondary education: AOR: 4.75, 95% CI: 0.47-8.17; Higher secondary and above: AOR: 8.52, 95% CI: 2.38-15.18) women are very likely to have ICT skills than illiterate primary educated women. Married women have more chances of having ICT skills than unmarried women. Also, unhappy women and lived in rural areas have a smaller amount of ICT skills than happy women who resided in urban areas. Most of the women who settled in other divisions have more odds compared to women who stayed in the Barishal division. Women who come from middle-class families (AOR:2.83, 95% CI: 0.05-2.36) hove the highest likelihood of ICT skills than women who come from the poorest families [**Table 2**].

4. Conclusion

This study targeted to examine the potential determinants for the status of media exposure and ICT skills among women in Bangladesh. The findings reveal that age, education, marital status, living area, geographical location, overall happiness and wealth status are significantly linked with the status of mass media exposure of women aged 15-49 years of Bangladesh. Since, the employment opportunities is gradually increasing in the ICT sector, and play as a potential factor for both national as well as international development so without its adoption, there will be a incredibly limited scope for development in both regional as well as national level. Therefore, government should design special plan and policies on infrastructural and financial supports and enhance the training facilities in school to encourage and improve the practice of ICT skills among girls besides women in urban and rural areas to become self-independent as well as self-confident which will be helpful to accelerate the achieving rate SDGs 4, 5 and 16 goals by ensuring 4.3, 4.4, 5.1, 5.b, and 16.10 targets.

Ethical Approval

This study used secondary data which is accessible through online upon a request from the data repository of MICS surveys as a result of ethical approval from our respective institutions was not required. The MICS taken all required ethical approval from the respective ethical review board prior to the collection of data.

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A Quantile Regression Analysis to Investigate the Effect of Temperature and Humidity on the Spread of COVID-19

Md. Rezaul Karim*
Department of Statistics, Jahangirnagar Universtiy, Savar, Dhaka-1342, Bangaldesh

Mst. Bithi Akter
Department of Statistics, Jahangirnagar Universtiy, Savar, Dhaka-1342, Bangaldesh

Abstract

The coronavirus disease 2019 (COVID-19) is an infectious disease caused by a newly discovered SARS-CoV-2 virus strain. The meteorological factors, especially temperature and humidity, are responsible for the spread of COVID-19. This research aims to analyze how temperature and humidity influence the spread of COVID-19 at different quantile levels. This study included 1365380 COVID-19 infected cases during the observation period (March 08, 2020 to August 12, 2021). To describe the spread of COVID-19, we consider the daily number of COVID-19 infected new cases as a response variable in the Negative Binomial regression analysis and the quantile regression analysis. The statistical shreds of evidence have shown that both temperature and humidity are highly significant on the lower and middle quantiles of the daily number of COVID-19 infected people. Besides, in the higher quantiles (for example, 90% quantile), the humidity is highly significant at any significance level, while the effect of temperature is insignificant on the spread of COVID-19.

Keywords: COVID-19, Negative Binomial Regression, Quantile Regression, Temperature, Humidity.

1. Introduction

Coronavirus is a significant pathogen affected by the severe acute respiratory system of humans. On December 31, 2019, an unspecified causal prevalence from Wuhan, Hubei, China, was reported to the World Health Organization (WHO). The novel disease later named COVID-19 has become a massive human health problem worldwide. The official name of COVID-19 is a SARS-CoV-2. The short name SARS-CoV-2 stands for severe acute respiratory syndrome coronavirus 2 (Li et al. (2020)). On March 11, 2020, the World Health Organization (WHO) announced COVID-19 as a pandemic because the number of infected new cases increased all over the globe and later proclaimed COVID-19 as a universal public health emergency (Sohrabi et al. (2020)). According to the report of Worldometers (2021) on August 18, 2021, the total COVID-19 infected cases are 209.59 million, and 187.86 million of among are recovered.

At first, two males and one female COVID-19 positive cases were detected on March 08, 2020, in Bangladesh. Since then, the COVID-19 has spread out day by day worldwide. The number of infected

* Corresponding author: rezaul@juniv.edu

people due to COVID-19 each day from March 03, 2020 to August 12, 2021, is presented in Figure 1.1. From Figure 1.1, it is observed that Bangladesh is already entered the third wave of coronavirus infections. Bangladesh effectively tackled the first wave of the spread of COVID-19 in 2020. The spread of the second wave is more significant than that of the first wave of the space of COVID-19. The third wave is now continuing, and it is hazardous compared to the previous waves on the report of UNB (2021). The number of deaths due to COVID-19 has reached its peak in Bangladesh, and new cases are being transmitted every day. A total of 1.45 million COVID-19 cases including 24891 deaths and 1.34 are recovered, are detected from March 08, 2020, to August 18, 2021 (Worldometers (2021)).

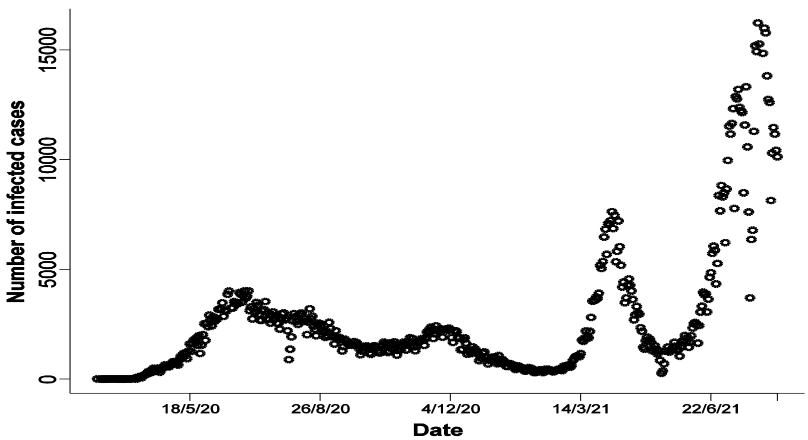

Figure 1.1. The Daily Number of Infected People Due to COVID-19 from March 03, 2020 to August 12, 2021 in Bangladesh.

The spread of COVID-19 may be affected by meteorological factors such as temperature and humidity. Still, many studies have been conducted to investigate the effects of temperature and humidity on the transmission of coronavirus disease. Mecenas et al. (2020) provided a systematic review paper on this issue. Most of the research showed that temperature and humidity affect the transmission of Coronavirus disease (see, for example, Karim et al. (2021)) used one-year data (up to the beginning of the second wave data) to inquire the effect of these two variables and found the significant impact of these variables on the daily average number of COVID-19 infected people in Bangladesh. However, they pointed out that the temperature and humidity alone do not describe most of the variations in COVID-19 diffusion. A few research found that there was no significant relationship between the spread of COVID-19 and the temperature (see, for example, Yao et al. (2020); Xie and Zhu (2020)).

The COVID-19 pandemic has not been over yet. The most recent wave, which is when there is a surge in the number of cases over a period of time in a particular region, has been documented in Haiti, Lesotho, and Sierra Leone, among other countries. While other factors contribute to waves, one currently is the appearance of virus variants, such as the highly infectious Delta variant detected in 92 countries as of June 21. The variant was first identified in India in October 2020 and continues to spread worldwide (Health (2021)). COVID-19 has continued to spread globally, and the next wave of COVID-19 appears probable. Thus the effect of meteorological factors on the spread of COVID-19 should be explored to help predict the progression over the upcoming months, although other factors may affect the evolution of the COVID-19 pandemic.

The main contribution of this article is to examine whether the temperature and humidity significantly affect the spread of COVID-19. Specifically, we focused on investigating the relationship between the arbitrary quantile of the daily number of infected people and some meteorological variables such as temperature and humidity after the second wave of the COVID-19 pandemic in Bangladesh. In addition, we also investigated whether those meteorological variables affect the daily average number of infected people. The impacts of the temperature and humidity on the conditional mean on the daily number of infected cases are investigated by applying the Negative Binomial regression analysis. In contrast, the impacts of those covariates on the conditional quantile of the daily number of infected cases are investigated by performing the quantile regression analysis. A research methodology including data source and data analytical tools are discussed in Section 2. Data exploration and statistical model-based evidence are presented in Section 3. The concluding remarks with a discussion are described in the final Section 4.

2. Research Methodology

2.1 Data Source

As a representative of the government of Bangladesh, the Institute of Epidemiology Disease Control and Research (IEDCR), Dhaka, Bangladesh, reports data of COVID-19 cases regularly, and these data are available in the link *http://corona.gov.bd/***.**The daily temperature (measured in ℃) and humidity (%) of Bangladesh are collected from Bangladesh Meteorological Department (BMD), Agargaon (Dhaka), Bangladesh. These data are also available in the web link *https://www.timeanddate.com/weather/bangladesh***.**

2.2 Regression Analysis

Suppose $(Y_1, \boldsymbol{X}_1), \dots, (Y_n, \boldsymbol{X}_n)$ be a random sample of size n drawn from $(Y, \boldsymbol{X})$ and the linear regression model can be written as follows:

$$Y_i = \boldsymbol{X}_i^T \boldsymbol{\theta} + \varepsilon_i \quad ; \quad i = 1,2, \dots, n, \tag{2.1}$$

where $\boldsymbol{X}^T = (1, X_1, \dots, X_d)$ be a set of observations of $\boldsymbol{X}$ and $\boldsymbol{\theta} = (\theta_0, \theta_1, \dots, \theta_d)^T$ and ε_i is a random disturbance term in the regression model. A traditional regression (also called mean regression) is summarized the relationship between the average value of the response variable and a set of covariates by the estimation of the conditional mean function $E(Y_i|\boldsymbol{X}_i) = \boldsymbol{X}_i^T\boldsymbol{\theta}$ (See, for example, Gujrati et al. (2012). We use Negative Binomial regression to estimate the conditional mean function for the daily number of new infected cases. It is a generalization of the Poisson regression model, which relaxes the restrictive assumption that the variance is equal to the mean made by the Poisson regression model. The details of Negative Binomial regression can be found in Agresti (2013).

2.3 Quantile Regression Analysis

The quantile regression analysis shows the relationship between an arbitrary quantile of a response variable and a set of explanatory variables through a regression model. Quantile regression is an extension of mean regression used for any skewed response distribution and bell-shaped distribution (see, for example, Koenker (2005), Gijbels et al. (2021)). It is widely used when the assumptions of the traditional mean regression are not met. The quantile regression, introduced by Koenker and Bassett (1978), gives knowledge about the non-linear and asymmetric effects of covariates on different quantiles in the conditional response distribution. Median (50% quantile) regression is a particular case of quantile regression, usually preferable for the skewed data set. Although quantile regression analysis is most often used to model specific conditional quantiles of the response, the full potential lies in modelling the entire conditional distribution.

The ordinary least squares regression gives a partial description of a conditional distribution since only one characteristic of the response distribution is provided by mean (conditional) regression. In contrast, to get a complete characterization of the distribution is provided by the quantile (conditional) regression. Quantile regression has some advantages, and they are:

i. the quantile regression estimator minimizes an asymmetrically weighted sum of absolute random disturbances rather than the sum of squared random disturbances, and thus the estimated coefficient vector is not sensitive to outliers,

ii. a quantile regression model operates a linear programming representation and simplifies examination, and

iii. the quantile regression approach is an attractive solution when the conditional distribution does not have a bell shape (e.g., normal shape), such as an asymmetric, bulky-tailed, or truncated distribution (for more details, see Kang (2014)).

The basic quantile regression model specifies the conditional quantile as a linear function of explanatory variables. For the regression model (2.1), the βth $(0 < \beta < 1)$ quantile function is $Q_\beta(Y_i|\boldsymbol{X}_i) = \boldsymbol{X}_i^T\boldsymbol{\theta} + Q_\beta(\varepsilon_i|\boldsymbol{X}_i)$.

For parameter estimation, it is assumed that the βth quantile of ε given $\boldsymbol{X}$ is zero. That is, for ith $(i = 1, \dots, n)$ observation, $Q_\beta(\varepsilon_i|\boldsymbol{X}_i) = 0$. Under this assumption, we can write $Q_\beta(Y_i|\boldsymbol{X}_i) = \boldsymbol{X}_i^T\boldsymbol{\theta}$ and the estimated βth quantile line is

$$Q_\beta\widehat{(Y_i|\boldsymbol{X}_i)} = \boldsymbol{X}_i^T\widehat{\boldsymbol{\theta}},$$

where $\widehat{\boldsymbol{\theta}}$ measures the marginal change in the βth quantile of Y_i due to a marginal change in $\boldsymbol{X}_i$.

It is very important to mention that the regression coefficient of the quantile regression model satisfies the following equivariant property, which means that the quantiles of the transformed random variable $h(Y)$ are simply the transformed quantiles on the original scale. This property is very useful for analyzing counts and survival data analysis (see details in Koenker (2005)).

2.3.1 Estimation of Quantile Regression Coefficient

The βth quantile regression coefficient estimator of $\boldsymbol{\theta}$ of the Model (2.1) can be estimated by minimizing the tick-loss function (see, Koenker and Bassett (1978)). That is, the βth quantile regression coefficient estimator is

$$\widehat{\boldsymbol{\theta}}(\beta) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_{i=1}^{n} \rho_\beta(Y_i - \boldsymbol{X}_i^T\boldsymbol{\theta}), \tag{2.2}$$

where $\rho_\beta(u) = |u|[(1-\beta)I(u \le 0) + \beta I(u > 0)]$ is a tick loss function, a special case of power tick loss function is provided by Gijbels et al. (2019). The tick loss function is also called the check-loss function. The Linear Programming Problem (LPP) is used to solve the problem. The linear regression model (2.1) can be rewritten as

$$\begin{aligned} Y_i &= \boldsymbol{X}_i^T\boldsymbol{\theta} + \varepsilon_i \\ &= \boldsymbol{X}_i^T\boldsymbol{\theta} + (u_i - v_i) \end{aligned}$$

where $u_i = \varepsilon_i I(\varepsilon_i > 0)$, and $v_i = |\varepsilon_i| I(\varepsilon_i < 0)$. Therefore, the new objective function for the βth quantile estimator is

$$\min_{\text{all} u_i, v_i} \beta \sum_{i=1}^{n} u_i + (1-\beta) \sum_{i=1}^{n} v_i$$

subject to

$$\begin{aligned} &Y_i = \boldsymbol{X}_i^T \boldsymbol{\theta} + u_i - v_i; i = 1, \cdots, n, \\ &\boldsymbol{\theta} \in R^{d+1}, u_i \geq 0, v_i \geq 0. \end{aligned}$$

Let $\boldsymbol{\phi} = [\boldsymbol{\theta}]_+$ and $\boldsymbol{\varphi} = [-\boldsymbol{\theta}]_+$, where $[z]_+$ is the nonnegative part of z. After some calculation, the LPP can be translated as

$$\left.\begin{aligned} &\text{minimize} \quad c^T \boldsymbol{x} \quad \text{with respect to} \quad \boldsymbol{x} \\ &\text{subject to} \quad \boldsymbol{Ax} \geq \boldsymbol{b} \\ &\ \boldsymbol{x} \geq \boldsymbol{0} \end{aligned}\right\} \quad \text{Primal Problem,}$$

where

$$\left.\begin{aligned} &c = [\boldsymbol{0}^T_{(d+1)\times 1} \boldsymbol{0}^T_{(d+1)\times 1} \quad \beta \boldsymbol{1}^T_{n\times 1} (1-\beta) \boldsymbol{1}^T_{n\times 1}]^T \\ &x = [\boldsymbol{\phi}^T_{(d+1)\times 1} \boldsymbol{\varphi}^T_{(d+1)\times 1} u^T_{n\times 1} v^T_{n\times 1}]^T \\ &A = [\boldsymbol{X}^T_{n\times(d+1)} \quad - \boldsymbol{X}^T_{n\times(d+1)} \boldsymbol{1}_{n\times n} \quad - \boldsymbol{1}_{n\times n}] \\ &b = [\boldsymbol{Y}_{n\times 1}]. \end{aligned}\right\}$$

Now, this LPP can be solved by Simplex Method, Frisch-Newton Interior Point Method, Interior Point Method with Prepossessing, Spare Regression Quantile fitting Method, etc. Each method has some merits and demerits. Details can be found in Koenker (2005). All methods are available in the package named *quantreg* in R software. Other standard software such as SAS and Python can also solve the above LPP. A friendly discussion about the quantile regression can be found in Koenker (2005). For statistical inference for the quantile estimation, we can use bootstrapping for estimating all unknown quantities with the standard error and hence p-value. Bootstrap is preferable because it does not assume the distribution of Y_i or ε_i. We use a nonparametric bootstrap procedure to estimate standard error and p-value, which is a default method in the *quantreg* package in R Software (See, Koenker (2009)).

3. Statistical Analysis and Results

3.1 Data Exploration

This section describes the basic features for a given dataset to make better investing decisions about the characteristics of a variable that depicts in features and tabular format. The total number of days is 515. Table 3.1 describes summary statistics of daily COVID-19 infected new cases and meteorological factors, especially temperature and humidity.

Table 3.1. Descriptive Statistics of the Daily Number of Infected New Cases Due to COVID-19, Temperature and Humidity from March 03, 2020 to August 12, 2021.

Variables	Minimum	Maximum	Mean (SD)	Skewness	Kurtosis
Infected new cases	0	16230	2651.223(3034.399)	2.399	8.842
Temperature	8	39	29.031(4.782)	-1.031	4.626
Humidity	21	10	64.687(17.344)	-0.356	2.474

The minimum daily number of COVID-19 infected new cases is zero, while the maximum is 16230. The average daily number of COVID-19 infected new patients is 2651. In addition, other elements indicate that the minimum temperature of 8 ℃ with the highest temperature of 39 ℃ and the minimum humidity of 21% with the maximum humidity of 100%.

The skewness statistic, measure the degree of asymmetry, of infected new cases is greater than one and positive. It means that the response variable COVID-19 infected new cases have a skewed distribution. See the detailed results in Table 3.1. Positive skewness refers to the size of the tail on the right side being more extensive than that on the left side. This is also confirmed by Figure 3.1. The kurtosis statistic, measure the relative peakedness or fatness of a distribution compared with the normal distribution, of infected new cases is greater than 3. It indicates that the data set has heavier tails than a normal distribution (more in the tails). In this case, if we use a classical regression model of the outcome variable log(number of infected new cases) due to COVID-19 for finding the effect of covariates, it might produce misleading results because the normality assumption is not met. The quantile regression, which does not need a normal distribution, might be an excellent alternative to deal with this problem.

The scatter plot of the daily number of infected new cases and the temperature with the LOWESS (Locally Weighted Scatterplot Smoothing) curve is depicted in 3.2 (a). The daily number of infected new cases and the humidity with the LOWESS curve is also presented in 3.2 (b). From Figure 3.2, it is observed that the daily number of COVID-19 infected new cases is positively correlated with the temperature and humidity. The LOWESS curves in both figures seem to be a positive trend that indicates the average number of infected people due to COVID-19 has increased for increasing the temperature and humidity. Later on, this is also confirmed by the results (See Table 3.2) of negative binomial regression analysis.

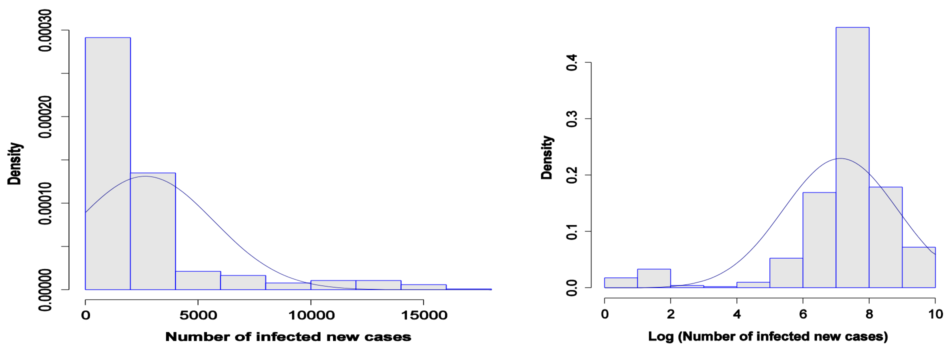

Figure 3.1. Histogram with Kernel Density Function (a). Number of infected new cases; (b). Log (Number of Infected New Cases) Due to Coronavirus Disease.

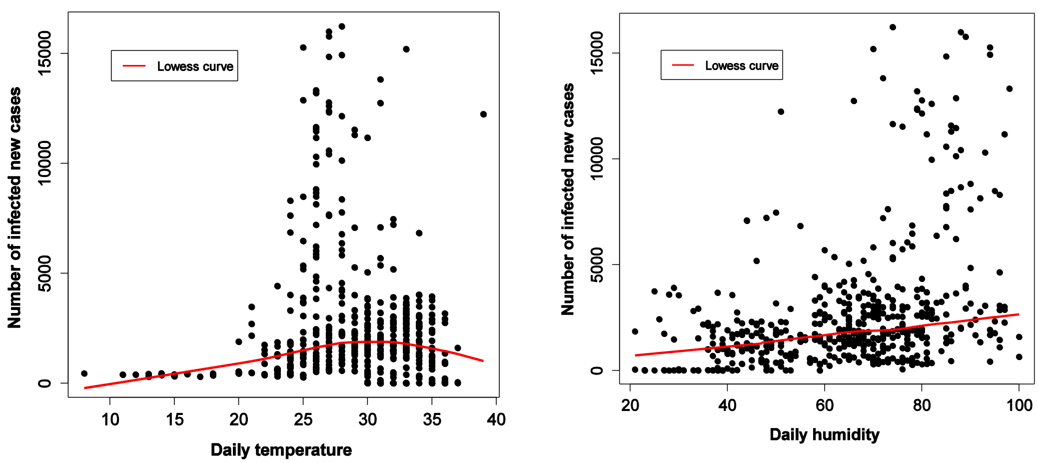

Figure 3.2. Scatter plot (a). Number of infected new cases due to COVID-19 vs. Temperature from March 03, 2020- August 12, 2021; (b). Number of Infected New Cases Due to COVID-19 vs. Humidity from March 03, 2020- August 12, 2021.

3.2 Negative Binomial Regression Analysis

Since our response variable named "number of infected new cases" due to COVID-19 is a count variable and does not have an excessive number of zeros, the Poisson regression model is a natural choice for finding the effect of covariates on the conditional mean of the daily number of infected new cases due to COVID-19. However, we found overdispersion in the Poisson regression model due to the ratio of the residual deviance by the degrees of freedom is much larger than one. The likelihood ratio test (LRT) for overdispersion also confirmed that the data are overdispersed (p-value < 0.001). The LRT is computed to compare a fitted Poisson model against a fitted Negative Binomial model. This test is available in the *DCluster* package in R Software. For this overdispersed data, the Poisson model does not well to the data. In that case, the next preferable model is a Negative Binomial regression model or Quasi-Poisson regression model that might fit better to the data. We use Negative Binomial regression, and the summary results of the Negative Binomial regression model are tabulated in Table 3.2.

Table 3.2. The Model-Based Result of the Estimated Negative Binomial Regression Model Given in (2.1)

Estimate	Coefficients	Std. Error	z-value	p-value
Intercept	4.541	0.401	11.319	<0.001
Temperature	0.047	0.010	4.537	< 0.001
Humidity	0.029	0.003	10.244	<0.001

The result shows that the temperature and humidity significantly affect the average number of infected new cases due to COVID-19, which supports the findings of Karim et al. (2021)). However, our main objective is to trace the effect of temperature and humidity on the different quantile levels of the number of infected people while investigating the mean effect of those variables on the response. Therefore, we consider quantile regression analysis proposed by Koenker and Bassett (1978) for further analysis.

3.3. Quantile Regression Analysis

The theory of quantile regression proposed by Koenker and Bassett (1978) is required a real-valued response variable. Therefore, the daily number of infected cases is not directly useful for quantile regression analysis mentioned in Section 2. Many researchers consider the logarithmic transformation of the "number of infected new cases" in the mean regression analysis (see, for example, Mecenas et al. (2020) and Karim et al. (2021)). We also select the logarithmic transformation of "number of infected new cases" due to COVID-19 to transform a real-valued response variable for the quantile regression analysis. Since the quantile regression estimators share the equivariance properties mentioned in Section 2, we add 1.1 to the response variable before the logarithmic transformation to avoid the log-scale transformation of zero. This transformation of the response variables with 1.1 added value does not affect the final results (see Koenker (2005)).

In quantile regression analysis, we focus on estimating the conditional quantile of the response variable $Y = \log(\text{number of infected new cases})$ given temperature and humidity by using the following regression model.

$$Y_i = \theta_0 + \theta_1 \times \text{temperature}_i + \theta_2 \times \text{humidity}_i + \varepsilon_i, \text{ for } i = 1,2,\dots,515 \tag{3.1}$$

This model is a particular specification (i.e., $d = 2$) of the regression model (2.1). In this case, we assume the $\beta$the quantile of ε_i given $\boldsymbol{X}_i = \{1, \text{temperate}_i, \text{humidity}_i\}$ is zero. That is $Q_\beta(\varepsilon_i|\boldsymbol{X}_i) = 0$.

Since the regression coefficient $\boldsymbol{\theta} = (\theta_0, \theta_1, \theta_2)^T$ of Model (3.1) for βth quantile estimation depends on the value of β, under the assumption of $Q_\beta(\varepsilon_i|\boldsymbol{X}_i) = 0$, the estimated βth conditional quantile based on model (3.1) can be written as, for $i = 1,2, \dots ,515$,

$$\widehat{Q_\beta(Y_i|\boldsymbol{X}_i)} = \hat{\theta}_0(\beta) + \hat{\theta}_1(\beta) \times \text{temperature}_i + \hat{\theta}_2(\beta) \times \text{humidity}_i, \tag{3.2}$$

where the estimated regression coefficient $\widehat{\boldsymbol{\theta}}(\beta) = (\hat{\theta}_0(\beta), \hat{\theta}_1(\beta), \hat{\theta}_2(\beta))^T$ is obtained by using the methodology given in Section 2. We now can estimate any conditional quantile of Y by using Equation (3.2); we only tabulated the summary results of five quantiles, namely 10%, 25%, 50%, 75%, and 90% quantiles of dependent variables in Table 3.3. The result shows that, at 10%, 25% and 50% quantiles are highly significant for both covariates: temperature and humidity. However, at 75% and 90% quantiles, the humidity is highly significant while the temperature is not.

Table 3.3. Summary Statistics of the Quantile Regression Equation for $\beta = (0.10,0.25,0.50,0.75,0.90)$ Given in (3.2)

	log (Number of infected new cases +1.1)			
Quantile	Estimate	$\widehat{\boldsymbol{\theta}}(se(\widehat{\boldsymbol{\theta}}))$	t-value	p-value
0.10	Intercept	-2.943 (1.210)	-2.432	0.015
	Temperature	0.051 (0.022)	2.378	0.018
	Humidity	0.100 (0.013)	7.554	<0.001
0.25	Intercept	2.004 (0.716)	2.797	0.005
	Temperature	0.071 (0.006)	11.031	<0.001
	Humidity	0.041 (0.009)	4.256	<0.001
0.50	Intercept	4.284 (0.462)	9.274	<0.001
	Temperature	0.062 (0.016)	3.908	<0.001
	Humidity	0.021 (0.003)	6.832	<0.001
0.75	Intercept	6.123 (0.673)	9.099	<0.001
	Temperature	0.014 (0.020)	0.699	0.484
	Humidity	0.041 (0.009)	4.256	<0.001
0.90	Intercept	5.811 (1.152)	5.043	<0.001
	Temperature	0.027 (0.034)	0.774	0.439
	Humidity	0.031 (0.005)	6.529	<0.001

Specifically, the estimated quantile regression model by using quantile regression equation (3.2) for 50% conditional quantile is for $i = 1,2, \dots ,515$,

$$\widehat{Q_{0.50}(Y_i|\boldsymbol{X}_i)} = 4.284 + 0.062 \times \text{temperature}_i + 0.021 \times \text{humidity}_i,$$

which is actually an estimated median regression equation because median regression is a 50% quantile of quantile regression.

For a different value of β $(0 < \beta < 1)$, the p-value for the regression coefficient of temperate and humidity of the quantile regression Equation (3.2) are also depicted in Figure 3.3. From the second plot of Figure 3.3, it is seen that all p-values are less than 0.05, which indicates that the humidity highly significantly affects the spread on COVID-19 at any quantile level. On the other hand, the temperature significantly affects the conditional quantile of the number of infected cases only for the value of $\beta \in (0.09, 0.71)$, which means that the temperate does not significantly affect lower and higher conditional quantile holding humidity is fixed.

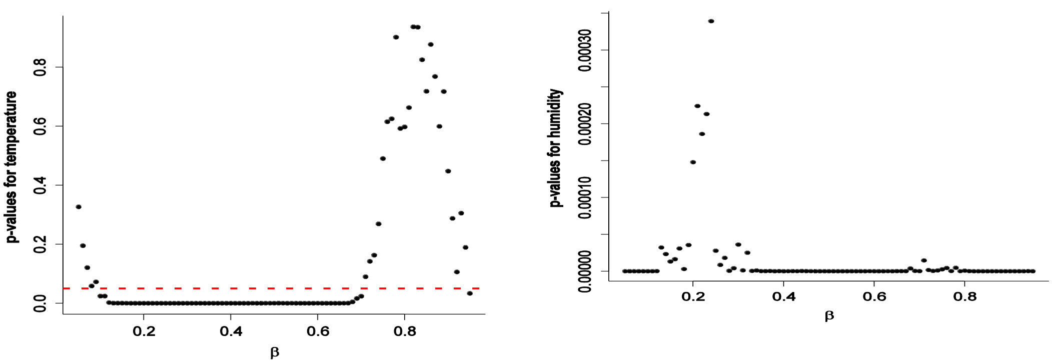

Figure 3.3. Using the Regression Model (3.1), (a). the p-value for Testing the Effect of Temperature against the Value of β; (b). the p-value for Testing the Effect of Humidity against the Value of β.

Figure 3.4 shows the trend of intercept, temperature and humidity coefficient on the number of infected new cases by quantiles. In particular, each panel represents an independent variable in the model; the horizontal axes display the quantiles while the estimated effects are portrayed on the vertical axes. The horizontal solid line parallel to the x-axis corresponds to the least-square coefficient along with the 95% confidence interval. Each dot represents the slope coefficient for the quantile shown on the x-axis. Therefore, the solid polygonal path shows the quantile regression pattern estimates along with the confidence bands.

4. Discussion and Concluding Remarks

This study's main objective was to investigate whether temperature and humidity increase or reduce the transmission of COVID-19. And if it helps to transmit disease, then whether it significantly affects the disease transmission or not. For this, we have collected daily COVID-19 infected data from IEDCR and BMD, Bangladesh. The number of daily new infected cases due to COVID-19 is considered a response variable, and the temperature and humidity are considered covariates for statistical analysis. Initially, we explore the data by descriptive statistics. Histograms with kernel density function have presented the shape of the response variable. We see that the response variable follows a skewed distribution. We also depicted the relationship between the response variable and covariates using a scatter diagram with a LOWESS curve. The scatter plots show the positive association between the number of infected people due to COVID-19 and temperature and humidity. The LOWESS curves also show that the average number of infected people due to COVID-19 has increased for increasing the temperature and humidity.

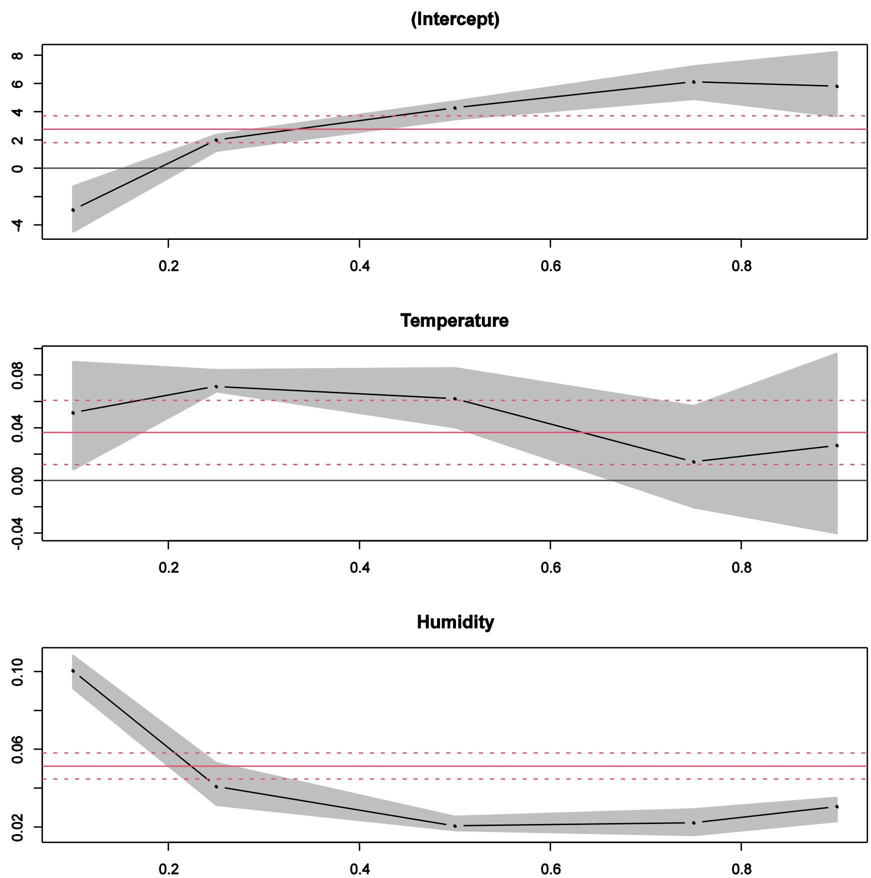

Figure 3.4. The Trend of Coefficient of Exploratory Variable, on Response Variable by Quantiles.

Since the number of infected new cases is a count variable and overdispersed, we fitted a Negative Binomial regression model. This model is mainly used to investigate the relationship between the average number of daily infected cases and the covariates: temperature and humidity. It is observed that both of the covariates significantly affect the spread of COVID-19. Increasing both variables leads to an increase in the spread of COVID-19. Many research findings have supported these findings (see, for example, Mecenas et al. (2020)). Karim et al. (2021) used a flexible fractional polynomial model and a B-spline model with a smoothing function of time to determine the effect of these covariates. They found a significant impact of both covariates in the flexible fractional polynomial model. In contrast, the flexible B-spline model significantly affected humidity but not considerably impact the temperature. Many types of research support their findings (see, for example, Mecenas et al. (2020)).

However, we are not only interested in the effect of the mean of responses in this study. We also focus on the relationship between the different quantile levels of the response variable and meteorological variables such as temperature and humidity. The mean regression analysis based on the Negative Binomial model only investigates the effect of covariates on the average effect of the response variable instead of the quantile levels of the response variable. Quantile regression analysis was performed to find the impact of those covariates on the quantile levels of the response variable. The summary results of the effect of covariates on 10%, 25%, 50%, 75% and 90% quantile of the response were tabulated in Table 3.3. The result indicates the significant relationship between some quantiles of the response variable and both covariates: temperature and humidity. Specifically, they positively impact the spread of COVID-19, but the lower and middle quantiles have significantly reduced the spread of COVID-19. This is also illustrated in Figure 3.4. However, there is a positive and insignificant effect of temperature on 75% and 90% quantiles of the response. These are also confirmed by Figure 3.3.

Note that the results of mean and quantile regression analyses are not directly comparable when the response variable is highly skewed (see, for example, Koenker (2005)). We can only compare some negative binomial regression analysis findings and 50% quantile regression (e.g., median regression) analysis findings when response distribution is very close to symmetric. However, in our analysis, both models showed that the temperature and humidity positively affect the spread of COVID 19 in Bangladesh.

Although we consider only two meteorological factors: temperature and humidity that significantly affect disease transmission, other factors such as public awareness, health policy, immunity system, human lifestyle, mask-wearing tendency, weather and climate, airflow, etc., might also be significantly responsible for transmitting COVID-19. Future research will focus on determining other factors that are the leading causes of increasing and decreasing the spread of COVID-19.

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Changes of Temperature and its Projection over Bangladesh Using CORDEX South Asian Data

Afruza Sultana*
Assistant Meteorologist, Bangladesh Meteorological Department, Dhaka, Bangladesh

Md. Bazlur Rashid
Meteorologist, Bangladesh Meteorological Department, Dhaka, Bangladesh

Md. Abdul Mannan
Meteorologist, Bangladesh Meteorological Department, Dhaka, Bangladesh

Md. Monowar Hossain
Meteorologist, Bangladesh Meteorological Department, Dhaka, Bangladesh

Md. Tufazzul Hosen Arif
Department of Economics, Jagannath University, Dhaka, Bangladesh

Abstract

The aim of this work is to observe the changes of temperature, calculate the rate of temperature change and find out the future temperature scenario of Bangladesh from the Coordinated Regional Climate Downscaling Experiment (CORDEX) over South Asia domain. The focus is on Bangladesh in the 21st century, using data from two Coupled Model Intercomparison Project 5 (CMIP5) models CCCma-CanESM2 and CNRM-CERFACS-CNRM-CM5 developed by Indian Institute of Tropical Meteorology (IITM), India and Swedish Meteorological and Hydrological Institute (SMHI) respectively which are downscaled by RegCM4 and RCA4 as a regional environment model. The study contains station data with RegCM4 models and changes in onset based on mean temperature at the end of the 21st century relative to base period (1976-2005). The driving CMIP5 models CCCma-CanESM2 and CNRM-CERFACS-CNRM-CM5, the RegCM4 and the RCA4 models show higher future temperature changes for RCP8.5 as compared to RCP4.5. For both model, temperature trends are increasing and statistically significant at 95% and 99% level. Model simulated temperature and observed temperature are 2-4°C for this systematic Bias correction method is used to error correction. RCP4.5 (RCP8.5) of CCCma-CanESM2_RegCM4-4 and CNRM-CERFACS-CNRM-CM5_RCA4 temperature may rise 1.83°C (3.21°C) and 1.69°C (2.94°C) during (2021-2100). For RCP4.5, the near future and far future the increasing rate of temperature is 0.0077°C/ year (0.0134°C/ year) and 0.0008°C/year (0.0029°C/year) for CCCma-CanESM2_RegCM4-4. Also for RCP8.5 the near future and far future the increasing rate of temperature is 0.0115°C/ year (0.0002°C/ year) and 0.0005°C/year (0.0046°C/year) for CCCma-CanESM2_RegCM4-4 (CNRM-CERFACS-CNRM-CM5_RCA4).

Key words: Temperature Change, CORDEX, CMIP5, RegCM4, RCP.

1. Introduction

Global heating will be one of the most significant eco-friendly problems in the 21st century and the effects of global warming are worldwide (Chan, 2006). Bangladesh is one of the most susceptible

* Corresponding author: afruza.sultana.ju@gmail.com

country in the world for the response of global warming (Richard, 2012). Earth heating is mainly caused by the escalation of greenhouse gases of the troposphere (Houghton, 2002). The hypothesis that increased greenhouse gas absorptions may lead to an upswing in global temperatures first emerged in the 1960s (Peterson et al., 2008). The mainstream of climate experts are agree with the indication for anthropogenic global warming is robust (Rosenberg et al., 2010). Intergovernmental Panel on climate Change (IPCC) has conveyed in the Fourth Assessment Report that worldwide surface temperature enlarged 0.74 ± 0.18 °C during the 100 years ending in 2005 (IPCC, 2007). IPCC (2007) also noted that the rise of average yearly temperature will be 3.3 °C per century. A large numbers of researchers have been carried out on trend of temperature change in climatic parameters over Bangladesh. It is pointed out that temperature tendency has been changed (Chowdhury and Debsharma, 1992; Mia, 2003; Islam, 2007). The stated average yearly temperature of Bangladesh has increased during the period of 1895-1980 at 0.31°C over the past two decades (Parathasarathy et al., 1987; Divya and Mehritra, 1995). Using the data of 1961- 1990 Bangladesh has been estimated to increase yearly average maximum temperature of 0.4°C and 0.73°C by 2050 and 2100 respectively (Karmakar and Shrestha, 2000).

For the year 1956-2005, the IPCC has stated a worldwide warming tendency of 0.13°C/decade (IPCC, 2007), and the update fifth assessment report (AR5), it was updated to 0.12°C per ten years for the period 1951-2021(IPCC, 2013). This temperature heating and related weather change unquestionably has long-term concerns for many environmental fields, such as drinking water, power sector, animal health condition, bio-diversity and ecologies. These may be influential issues to the establishment of definite environmental pollutants linked with the rise in air temperature (Cueto et al., 2007, Shimoda et al 2003 & Stone et al, 2001). The increase of air temperature is likely to lead to a rise in air contaminants (Kalisa et al., 2018).

Bangladesh is located in the sub-tropical monsoon climate region. The climatology of Bangladesh are described the following four season: 1. Northeast monsoon (December- February) or winter, 2. Pre-monsoon (March-May) or summer 3. Southwest monsoon (June-September) and 4. Post-monsoon or autumn. In winter or Northeast monsoon the mean temperature range is 18-22°C. Average temperature in January differ from about 17°C in the northwestern regions and northeastern parts to 20-21°C in the coastline belts. In pre-monsoon season the mean temperature varies between 23-30°C. The highest maximum temperature ranging from 36-40°C is attained in the northwestern and southwestern districts of the country. Average temperature in July varies from about 27°C in the southeast to 29°C in the northwestern regions of Bangladesh. Temperature falls noticeably but the lowest minimum does not generally fall below than 10°C throughout the country (Khatun et al., 2016).

Technology based weather or climatic evidence for both near and more far future period is required to contribution with long-term development. In this linking, a global climate model (GCM) or Regional

Climate Model (RCM) is the only device that can produce future climate outputs (Almazroui et al., 2016). The IPCC AR5 report used the CMIP5 multi-model information-base established under the World Climate Research program. Downscaling is generally used to convert the GCM productivities into a better right arrangement (Lanzante et al., 2018). Dynamic downscaling method is built on physical process which are in fact the RCMs. Statistical downscaling is another technique constructed on empirical nature where downscaled projections persist fixed over time (Dixon et al., 2016). There are advantages and disadvantages in both dynamical and statistical downscaling techniques. The statistical downscaling is easy and ingests less computing memories. Because our goal is to realize the weather or climate factors, the use of dynamical downscaling is favored for this investigation where different micro physics schemes can be selected for improved results. The CMIP5 GCMs are generally low regulation (100-300 km) models and are not appropriate for producing comprehensive climatic parameters over a precise locations (Murphy et al., 1995). GCMs cannot regenerate the full configuration of location specific climatic data because they have no geographic data in better scales. To solve this issues, RCMs are deliberated to be the best methods for downscaling GCM-outputs climatic structures, to achieve more exhaustive climate results over a precise areas (Giorgi et al., 2001 & Jones et al., 2004). RCMs are also precious over locations where station data are either scare or absent, as over the South Asia. The RCMs overtake the driving GCM and deliver extra value to regenerate the climate of an area (Almazroui, 2012). Consequently, dynamic downscaling of GCM model has been a broadly used and satisfactory approach (Christesen et al., 2003 & Pal et al., 2007). On the other hands, an RCM can be applicable to generate future climate products as well as to support recognize the past climate in actual area. Therefore, the climate information produced by appropriate RCM can be used in analysis in the South Asian area, aiming on Bangladesh for the projection at future. The goal of this research to examine the deviations in temperature over the CORDEX-South Asia domain (20.79°W – 138.09°E and 42.42°N – -25.23°S) with an attention on the Bangladesh during the 21st century and is used information from GCMs from CMIP5 simulated with a RCM system updated in 2010 (RegCM4). With the intention of understand the probable changes in temperature over Bangladesh, the corrected or amended temperature is projected into the future.

2. Model, Data and Methodology

2.1 Model and Data

Regional Climate Model version 4 (RegCM4) (Giorgi et al., 2012) are used for CORDEX downscaling by the International Center Theoretical Physics (ICTP). This model consider the following objects: 1. Land surface, 2. planetary boundary layer, 3. Air- sea flux scheme, 4. Cumulus convection, 5. Radiative transfer, 6. Interactive aerosols, 7. Interactive lake, 8. Tropical band, 9. Coupled ocean, 10. A gas phase chemistry module (CBM-Z), 11. Resolve scale precipitation, 12. Dynamics (Hydeosatic, σ- vertical coordinate).

Rossby Centre regional atmospheric model version 4(RCA4) (Samuelsson et al., 2015) are used for the CORDEX downscaling. Rossby Centre regional atmospheric model version 4(RCA4) (Samuelsson et al., 2015) are used for the CORDEX downscaling. In this model: 1. Physiography data, 2. the number of soil layers with respect to soil moisture and soil columns with respect to soil water under forest and open land, 3. the density of organic carbon 4. An exponential root distribution 5. The prognostic snow albedo, 6. The dynamic vegetation model LPJ-GUESS, 7. Global lake-depth data base are considered.

The second generation Canadian Earth System Model (CanESM2) is the fourth generation coupled global climate model developed by Canadian Centre for Climate Modelling and Analysis (CCCma) of Environment and Climate Change Canada. Coupled Model Intercomparison Project 5 (CMIP5) models CCCma-CanESM2 compose an atmosphere-ocean general circulation model, a land-vegetation model and terrestrial and oceanic interactive carbon cycle. It simulates well the observed 20ths century Arctic temperature variability that includes the early and late 20^{th} century warming periods and the intervening 1940-1970 period substantial cooling. The spatial resolution of this model is 2.81°. CCCma-CanESM2 is derived by Center for climate change Research (CCCR), IITM, India.

CNRM-CERFACS-CNRM-CM5 model has been jointly developed by National Center for Meteorological research –Meteorological Atmosphere Studies Group and European Center for Research and Advanced Training in order to contribute phase 5 of the Coupled model Intercomparison Project (CMIP5).This model includes the atmospheric model ARPEGE-Climate, the ocean model NEMO, the land surface scheme ISBA and the sea ice model GELATO coupled through the OASIS system. The spatial resolution of this model is 1.41°. CNRM-CERFACS-CNRM-CM5 is derived by SMHI at Sweden.

For all the RCMs, CORDEX use a similar domain and having similar resolution of .44(~50 km). Here 27°N-93°E and 20°N-87°E is select for Bangladesh from the RCM domain of South Asia Region 42.42°N-138.09°E and -25.22°N-20.79°E. RegCM4 and RCA4 simulations cover the period 1976-2005 for the current day, with actual greenhouse gas absorptions, and 2006-2099 for future period with greenhouse gas absorptions following the representative concentration pathways RCP4.5 and RCP8.5. Observed temperature data are collected from Bangladesh Meteorological Department (BMD) during 1976-2005.

2.2 Methodology

2.2.1 Mann Kendall (MK) trend test

The non-parametric Mann-Kendall (MK) experiment (Kendall, 1975; Mann, 1945) is huge generally used for tendencies classifying in climatologic and weather data in time series analysis. Here, there are two benefits of using MK experiment. (1) It is a non-parametric test where data do not necessary to be normally distributed. (2) The trial has little sensitivity to unexpected due to inhomogeneous time series analysis (H. Tabari et al, 2011). The MK test statistic (S) is given by:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sign(T_j - T_i), \tag{1}$$

$$Sign(T_j - T_i) = \begin{cases} 1 \; if \; T_j - T_i > 0 \\ 0 \; if \; T_j - T_i = 0 \\ -1 \; if \; T_j - T_i < 0. \end{cases} \tag{2}$$

Where, T_j and T_i are the yearly values in years j and i, j > i respectively.

If n < 10, the value of $|S|$ is shares straight to the theoretical distribution of S derived MK. The two tailed experiment is used. At definite likelihood level H_0 is rejected in favor of H_1 if the original value of S equals or exceeds a certain value $S_{\alpha/2}$, where $S_{\alpha/2}$ is the lowest S which has the possibility less than α/2 to execute in case of no trend. A positive (negative) value of S designates an upward (downward) trend. For n ≥ 10, the statistic S is almost normally distributed with the mean and variance as follows:

$$E(S) = 0. \tag{3}$$

The σ^2 for the S statistic is expressed by:

$$\sigma^2 = \frac{n(n-1)(2n+5) - \sum t_i(i)(i-1)(2i+5)}{18}. \tag{4}$$

Where t_i represents the number of ties to extent i. The synopsis term in the numerator is used only if the data series contains tied values. The standard test statistic Z_s is considered as bellows:

$$Z_s = \begin{cases} \frac{S-1}{\sigma} \; for \; S > 0 \\ 0 \; for \; S = 0 \\ \frac{S+1}{\sigma} \; for \; S < 0. \end{cases} \tag{5}$$

The test statistic Z_s is used a quantity of meaning of trend. In detail, this experiment statistic is used to experiment the null hypothesis, H_0. If $|Z_s|$ is greater than $Z_{\alpha/2}$, where α denotes the special implication level (e.g., 5% with $Z_{0.025} = 1.96$) then the null hypothesis is inacceptable suggesting that the trend is important.

2.2.2 Sen's Slope estimator test

Non-parametric technique (Sen, 1968) was used to assessment the degree of tendencies in the data at time series analysis. The gradient of "n" pairs of statistics can be first projected by using the below equation:

$$\beta_i = \textbf{Median} \left[\frac{X_j - X_k}{j-k}\right] \; \forall \; (k < j) \tag{6}$$

In this formula, X_j and X_k represent values statistics at time j and k, separately, and time j is after time k (k ≤ j). The median of "n" values of β_i is the Sen's slope estimator experiment. A negative β_i value signifies a declining tendency; a positive β_i value signifies an accumulative tendency over time.

If "n" is an even number, then the slope Sen's estimator is computed by using the following equation:

$$\beta_{med} = \frac{1}{2}(\beta_{[n/2]} + \beta_{[(n+2)/2]}) \tag{7}$$

If "n" is an odd number, then the slope Sen's estimator is calculated by using the below formula:

$$\beta_{med} = \beta_{[(n+1)/2]} \tag{8}$$

Finally, β_{med} is verified by a two tailed experiment at 100(1-α) % assurance level, and the true gradient of monotonic tendency can be projected by using a nonparametric experiment (Partal and Kahya, 2006).

2.2.3 Systematic Bias Correction

2.2.3.1 Bias Correction for Original Model Simulation Temperature

For original model simulation temperature, in this bias correction method a constant corrected factor is estimated by the difference between average temperatures of original model simulations and observations for each month of base year. Then the constant corrected factor is subtracted from each month of every year.

Monthly bias:

$$BC_m = \overline{Ts_m} - \overline{To_m}\ . \tag{9}$$

Where, m = 1, 2…., 12 months, Ts and To are simulated & observed temperature and BC_m is corrected factor.

Corrected temperature base:

$$Tc_{tm} = Ts_{tm} - BC_m \tag{10}$$

Where, m = 1, 2…., 12 months and t = year

2.2.3.2 Bias Correction for Future Model Simulation Temperature

For future RCM simulation temperature, in this method a constant corrected factor is calculated by the difference between average temperatures of future model simulations and average temperatures of original model simulations for each month. After that the constant factor is subtracted from each month of every year.

Monthly bias:

$$BCF_{tm} = \overline{Tf_m} - \overline{Tc_{tm}} \tag{11}$$

Where, Tf_m & Tc_{tm} are future & original temperature of model simulation and BCF_m is corrected factor for future

Corrected temperature future:

$$Tcf_{tm} = Tf_{tm} - BCF_{tm} \tag{12}$$

Where, m = 1, 2…., 12 months and t = year

3. Results and Discussions

1. Seasonal Variation and Adjusted for Monthly Mean Temperature of Model Simulated Temperature of IITM and SMHI During 1976 – 2005

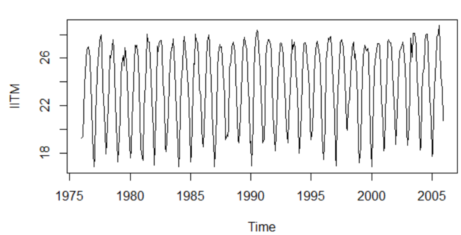

Figure 1a. Seasonal Variation of Monthly Mean Temperature for IITM During 1976-2005

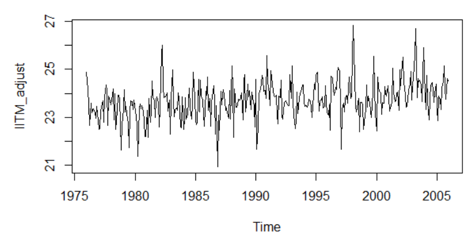

Figure 1b. Seasonally Adjusted of Monthly Mean Temperature for IITM During 1976-2005

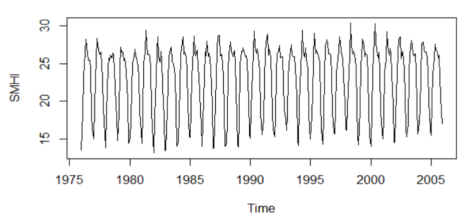

Figure 1c. Seasonal Variation of Monthly Mean Temperature for SMHI During 1976-2005

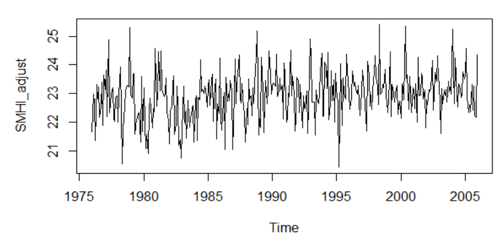

Figure 1d. Seasonally Adjusted of Monthly Mean Temperature for SMHI During 1976 -2005

Fig (1a &1c) show that both model contain seasonal variation and IITM model mean temperature variation is peak in May and trough in December of IITM model and also peak in May and trough in December of SMHI model each year and. Fig (1b & 1d) show that after removing seasonal variation trend is present.

Table 1. Seasonal Mann Kendal Test and Adjusted Value for Monthly Mean Temperature in Bangladesh During 1976-2005

Model	tau(τ)	2-sided p value	Seasonal factor	
			Largest	Lowest
CCCma-CanESM2_RegCM4-4 (IITM)	0.149	6.4258e-05	July 3.625	January -5.649
CNRM-CERFACS-CNRM-CM5_RCA4 (SMHI)	0.0633	0.072961	May 4.932	January -8.113

The test statistic of IITM model is 0.149 and the corresponding two-sided p-value is 6.4258e-05 and the test statistic of SMHI model is 0.0633and the corresponding two-sided p-value is 0.072961. So both model are rejected the null hypothesis and trend is present in both model (Table 1). The largest seasonal factor for IITM model is for July about 3.6251139 the lowest is for January about -5.6487290 which indicate that mean temperature is peak in May and trough in December each year & the largest seasonal factor for SMHI model is for May about 4.9322498 and the lowest is for January about -8.1125554 which indicate that mean temperature is peak in May and trough in December each year (Table 1).

2. Monthly Mean Temperature and Annual Mean Temperature of Simulated Historical Temperature for IITM and SMHI During 1976 – 2005

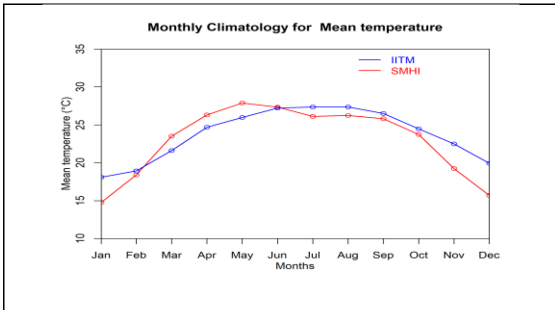

Figure 1e. Monthly Temperature Trend of IITM and SMHI Data for Base Year During 1976-2005

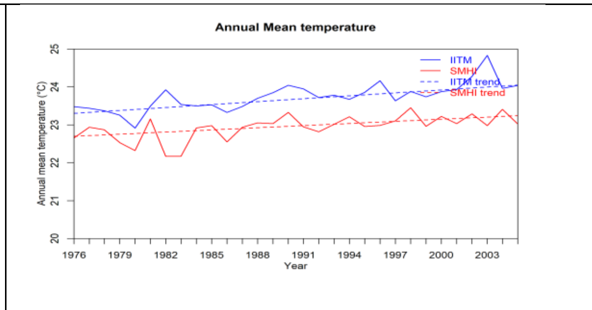

Figure. 2f. Annual Temperature Trend of IITM and SMHI Data for Base Year During 1976-2005

Monthly temperature cycle of simulated temperature during (1976-2005) shows that IITM (CCCma-CanESM2_RegCM4-4) is under estimated for March to June and rest of months are likely to above than SMHI (CNRM-CERFACS-CNRM-CM5_RCA4) in Bangladesh (Fig. 2a). Annual temperature trend over Bangladesh IITM and SMHI indicates both follows positive trend but IITM represent (1-2°c) higher than SMHI (Fig. 2b).

Table 2. Mann-Kendall experiment and Sen's slope estimator for mean temperature in Bangladesh During (1976-2005)

Model	First Year	Last year	tau(τ)	2-sided p value	Significance	Sen's Slope	SSmax95	SSmin95	SSmax99	SSmin99
CCCma-CanESM2_RegCM4-4	1976	2005	0.6	3.457e-06	Yes	0.025	0.0184	0.0367	0.0157	0.043
CNRM-CERFACS-CNRM-CM5_RCA4	1976	2005	0.457	0.00042	Yes	0.018	0.0056	0.0321	0.0038	0.0363

Trends of temperature are increasing and statistically significant over Bangladesh for the both models (Table 2). Temperature has changed each year 0.025°C during (1976-2005) for CCCma-CanESM2_RegCM4-4 model. It is also shown that temperature has changed each year 0.018°C during (1976-2005) for CNRM-CERFACS-CNRM-CM5_RCA4. For CCCma-CanESM2_RegCM4-4 model the upper and lower limit of the 95% confidence interval of Sen's slope estimate is 0.0184 & 0.0367 and the upper & lower limit of the 99% confidence interval of Sen's slope estimate is 0.0157 & 0.043. For CNRM-CERFACS-CNRM-CM5_RCA4 model the upper and lower limit of the 95% confidence interval of Sen's slope estimate is 0.0056 & 0.0321 and the lower limit of the 99% confidence interval of Sen's slope estimate is 0.0038 & 0.0363 in Table1.

1. Monthly Mean Temperature and Annual Temperature of Model Simulated Temperature for RCP 4.5 and RCP 8.5 (IITM and SMHI) During Near Future (2021-2050) and Far Future (2071 – 2021) over Bangladesh

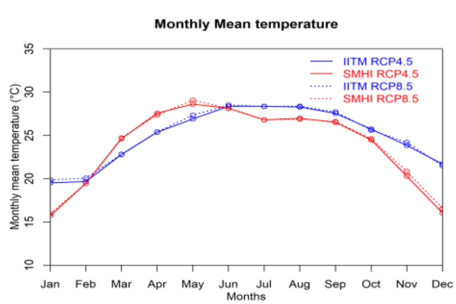

Figure 3a. Monthly Temperature Trend of IITM and SMHI Data for Base Year During (2020-2050)

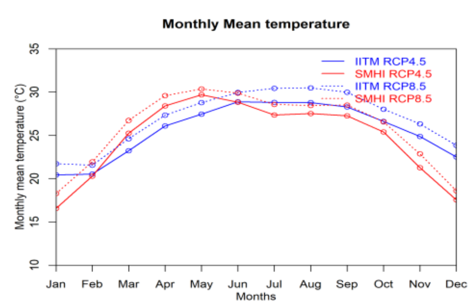

Figure 3b. Monthly Temperature Trend of IITM and SMHI Data for Base Year During (2071-2100)

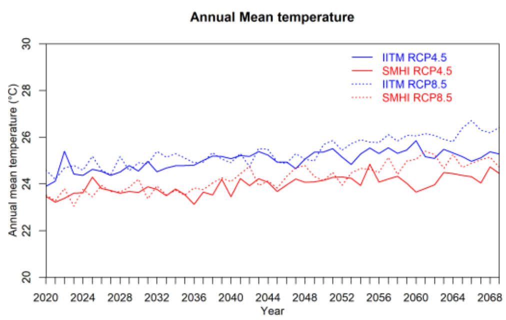

Figure 3c. Annual Temperature for Future Year of IITM RCP (4.5 & 8.5) and SMHI RCP (4.5& 8.5) During (2020-2050)

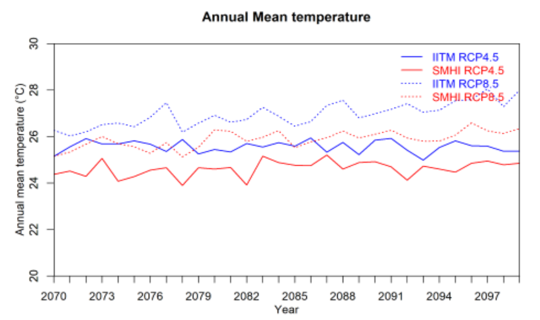

Figure 3d. Annual Temperature for Future Year of IITM RCP (4.5 & 8.5) and SMHI RCP (4.5& 8.5) During (2071-2100)

In Figs 3a-3b, in near future and far future, monthly mean temperature for both model of IITM and SMHI temperature scenario of RCP4.5 is less than RCP8.5 and monthly mean temperature of SMHI model simulated temperature is higher than the IITM model simulated temperature except March, April & May. Model simulated annual temperature of RCP4.5 and RCP8.5 display temperature scenario for both model temperatures of RCP4.5 is less than RCP8.5 (Fig 3c-3d). According to (Fig. 3d-3c) time series analysis of RCP4.5 and RCP8.5 for both model simulated temperature follows increasing trend and also specifies temperature of IITM (CCCma-CanESM2_RegCM4-4) is high than temperature of SMHI (CNRM-CERFACS-CNRM-CM5_RCA4).

Table 3. Temperature (°C) change for CCCma-CanESM2_RegCM4-4 (4.5 & 8.5) and CNRM CERFACS-CNRM-CM5_RCA4 (4.5 & 8.5) in Bangladesh from 2021-2100

RCP	Model	Change
RCP4.5	CCCma-CanESM2_RegCM4-4	1.83
RCP4.5	CNRM-CERFACS-CNRM-CM5_RCA4	1.69
RCP8.5	CCCma-CanESM2_RegCM4-4	3.21
RCP8.5	CNRM-CERFACS-CNRM-CM5_RCA4	2.94

In Table 3, it is shown that before bias correction the changes in future temperature, the RegCM4 simulated temperature for RCP4.5 of CMIP5 model CCCma-CanESM2_RegCM4-4(CNRM-CERFACS-CNRM-CM5_RCA4) is 1.83°C (1.69°C) and for RCP8.5 the changes of future temperature is 3.21°C (2.94°C) from 2021-2100 in Bangladesh.

4. Monthly Mean Temperature and Annual Mean Temperature of Observed and Model Simulated Temperature before and after Dias Correction During (1976-2005) of Bangladesh

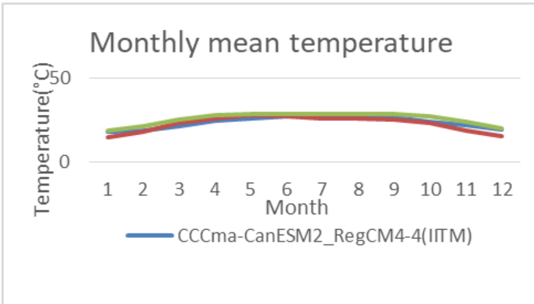

Figure 4a. Before bias correction Monthly Mean Temperature During (1976-2005)

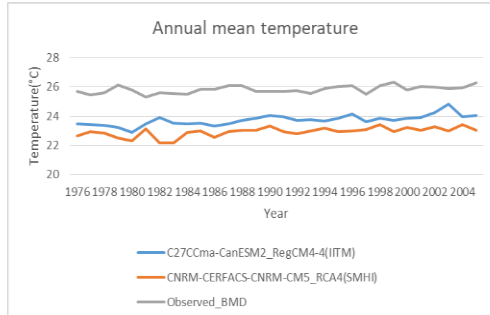

Figure 4b. Before Bias Correction Annual Mean Temperature During (1976-2005)

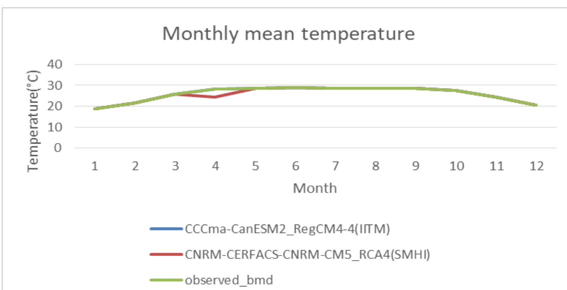

Figure 4c. After Bias Correction Monthly Mean Temperature During (1976-2005)

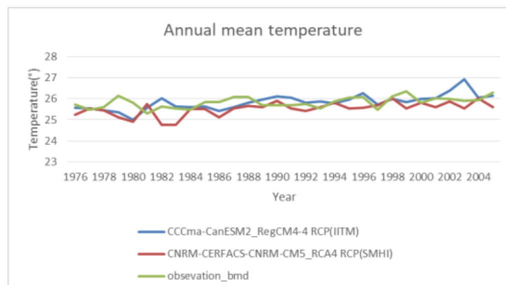

Figure 4d. After Bias Correction Annual Mean Temperature During (1976-2005)

Before bias correction for monthly mean temperature, the observed temperature is higher than the both model simulated temperature but temperature difference is very short (Fig 4a) and for annual mean temperature, the observed temperature is very high than the both model simulated temperature about (2-4) °C during (1976-2005) for Bangladesh (Fig 4b). After bias correction for monthly mean temperature, the observed temperature and the both model simulated temperature are similar without March, April & May of CCCma-CanESM2_RegCM4-4 model temperature (Fig 4c) and for annual mean temperature, the observed temperature and model simulated temperature is close with both model (Fig 4d).

5. Temperature Projection for RCP4.5 and RCP8.5 of CCCma-CanESM2_RegCM4-4 and CNRM-CERFACS-CNRM-CM5_RCA4 Models During 2021-2050 and 2071 – 2100 Over Bangladesh

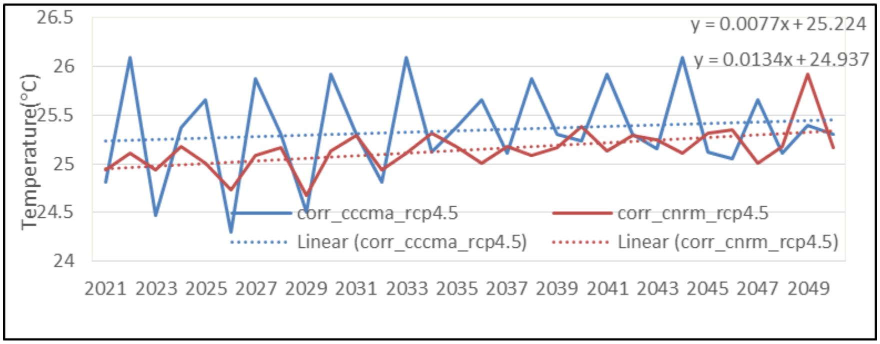

Figure 5a. Temperature Projection for RCP4.5 During 2021-2050

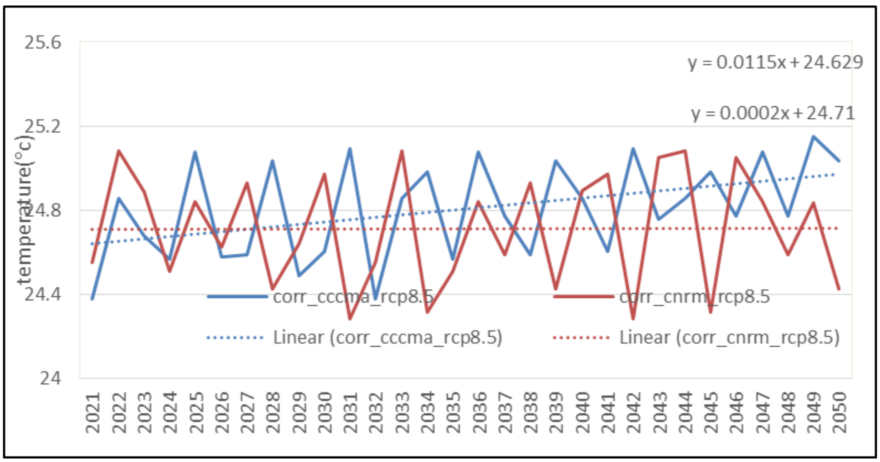

Figure 5b. Temperature Projection for RCP8.5 During 2021-2050

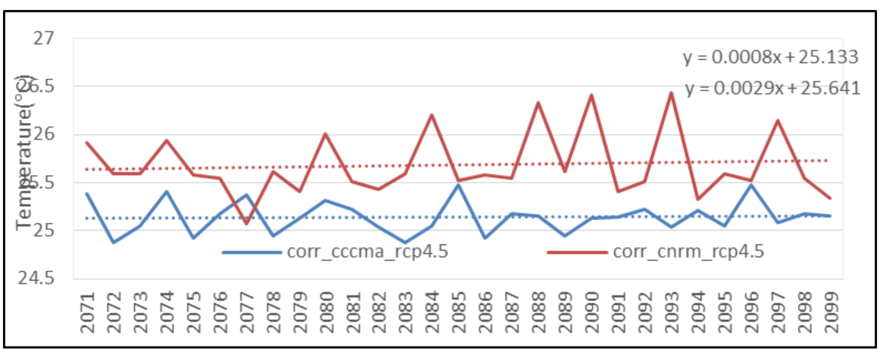

Figure 5c. Temperature Projection for RCP4.5 During 2071-2100

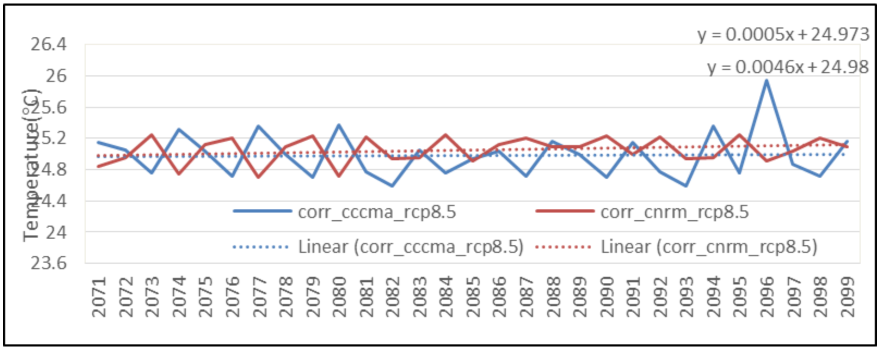

Figure 5d. Temperature Projection for RCP8.5 During 2071-2100

Table 4. Rate of temperature change in Bangladesh based on 1976-2005

Model	Emission scenario			
	RCP4.5		RCP8.5	
	near future	far future	near future	Far future
CCCma-CanESM2_RegCM4-4	0.0077	0.0008	0.0115	0.0005
CNRM-CERFACS-CNRM-CM5_RCA4	0.0134	0.0029	0.0002	0.0046

After the bias correction of RegCM4 simulated temperature for the near future (2021-2050) and the far future (2071-2100) for two scenarios (RCP4.5 and RCP4.8) of two CMIP5 model (CCCma-CanESM2_RegCM4-4 and CNRM-CERFACS-CNRM-CM5_RCA4) are shown in Fig 5a – Fig 5d and results are shown in Table 4. For RCP4.5, the near future and far future the mean temperature of CCCma-CanESM2_RegCM4-4 (CNRM-CERFACS-CNRM-CM5_RCA4) follows increasing trend and rate of temperature change is 0.0077°C/ year (0.0134°C/ year) and 0.0008°C/year(0.0029°C/year). Also for RCP8.5 the near future and far future the mean temperature of CCCma-CanESM2_RegCM4-4 (CNRM-CERFACS-CNRM-CM5_RCA4) follows increasing trend and rate of temperature change is 0.0115°C/ year (0.0002°C/ year) and 0.0005°C/year(0.0046°C/year).

3. Conclusions

In this paper, the trend of temperature in the 21st century over the CORDEX South Asian domain with a special effort on Bangladesh has been projected from RegCM4 model simulation. For both model, temperature trends are increasing and statistically significant level at 95% and 99%. Before bias correction of both climate models for monthly mean temperatures are under estimated by observed data, after bias correction only March, April and May are under estimate to CNRM-CERFACS-CNRM-CM5_RCA4. Annual mean temperature by climate models are significantly lower and temperature differences are (2-4) °C than observation, after bias correction model simulated data and observation are very close. IITM model projection for mean temperature is more comparable and its rate of change is higher than SMHI projection in past climate of Bangladesh during (1976-2005). For RCP4.5 (RCP8.5) of CCCma-CanESM2_RegCM4-4 and CNRM-CERFACS-CNRM-CM5_RCA4 temperature may rise 1.83°C (3.21°C) and 1.69°C (2.94°C) during (2100-2100). For RCP4.5, the near future and far future the increasing rate of temperature is 0.0077°C/year (0.0134°C/ year) and 0.0008°C/year (0.0029°C/year) for CCCma-CanESM2_RegCM4-4 respectively. Also for RCP8.5 the near future and far future the increasing rate of temperature is 0.0115°C/ year (0.0002°C/ year) and 0.0005°C/year (0.0046°C/year) for CCCma-CanESM2_RegCM4-4 (CNRM-CERFACS-CNRM-CM5_RCA4) respectively.

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HIV Estimation and Projection in Bangladesh up to 2030

Farhana Akter Bina
Department of Statistics, Jahangirnagar University, Savar, Dhaka, 1342, Bangladesh

Sabina Yasmin*
Department of Statistics, Jahangirnagar University, Savar, Dhaka, 1342, Bangladesh

Saima Khan
UNAIDS, Bangladesh

Mohammed Nazmul Huq
Department of Statistics, Jahangirnagar University, Savar, Dhaka, 1342, Bangladesh

Abstract

HIV epidemic models are mostly applied to describe the HIV epidemic in a proper way. It is inconvenient sometimes to enumerate the accurate number of individuals living with HIV, newly infected with HIV, or who have died of AIDS as a result of the response. With this backdrop, this paper attempts to estimate and project the HIV epidemic trend in Bangladesh. To analyze the epidemiological and demographical HIV/AIDS estimates and projections for the period 2018-2030 in the Spectrum software package, two models – AIDS Epidemic Model (AEM) and AIDS Impact Model (AIM) were used. Several nationally representative data sources and program data were used for the analysis. At the end of 2030, the number of PLHIV might reach up to 30743 which is 2.2 times higher than in 2018 where the estimated number of newly infected with HIV will be increased to 2,225 in 2030 from 1,596 which is nearly 1.5 times higher than in 2018. The percentage share of males in new infections showed a downward trend while females represent an upward trend from 2018 to 2030.The annual AIDS death will be eventually decreasing between 2018 and 2030. Among sub-population groups, the percentage of current HIV infections will significantly rise for the PWID and low-risk people mostly low-risk females. According to the mode of transmission, the principal source of HIV infection was needle sharing among adults (15+) in 2018 (37%), which will gradually decrease to 24% in 2030. It is indispensable to evaluate HIV prevention programs and guiding national HIV/AIDS policies in Bangladesh, by estimating HIV and regular updating of the estimates will improve the understanding of the HIV/AIDS epidemic.

***Keywords**:* Key population, People living with HIV (PLHIV), New HIV infections, HIV Deaths

1. Background

AIDS is a viral disease that has become known as 'the plague of the century' (Haghdoost, 2011). Globally the general trend of HIV is upward, however, in several countries the trend is downward. According to UNAIDS, an estimated 37.9 million people were living with HIV (including 1.7 million children) and 1.7 million new HIV infections were identified along with 1.1 million who died from AIDS (UNAIDS, 2019).

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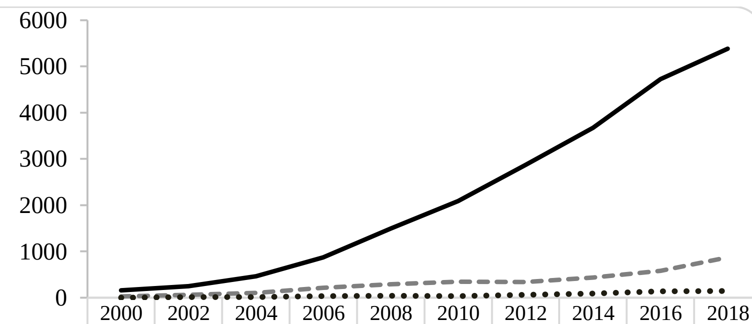
* Corresponding author: shila.ju588@gmail.com

The government of Bangladesh (GOB) introduced an early response to the HIV (Human Immunodeficiency Virus) epidemic in the mid1980s. In 1989, the first case of HIV was diagnosed in Bangladesh, thereafter, there was a trend for increasing numbers of infections being recorded (MoHFW, 2015). Even though Bangladesh has still been considered a low-prevalence country for the HIV/AIDS epidemic, with an overall prevalence in the general population of less than 0.01 percent (over the years) it poses a major threat to an epidemic due to the high prevalence of HIV in the neighboring country (Sultana, 2019). Bangladesh is one of the seven countries in the Asia and the Pacific region where HIV new infections continue to increase (Stover, *et al*., 2006). Every year on World AIDS Day, the Ministry of Health and Family Welfare (MoHFW) releases statistics on the number of people living with HIV (PLHIV) and deaths from AIDS in Bangladesh, and these figures have been continuously growing as shown in Figure 1. As of October 2018, there were cumulatively 6,455 HIV reported cases (including FDMN cases) of whom 1,072 have died. In 2018, the reported number of PLHIV was 5,383 (ASP, 2018), with an estimated 13,800 PLHIV (ASP & UNAIDS, 2019).



	2000	2002	2004	2006	2008	2010	2012	2014	2016	2018
Total number of reported PLHIV	157	248	465	874	1495	2088	2871	3674	4725	5383
Reported new cases	31	60	102	216	288	343	338	433	578	869
New reported deaths	1	16	14	35	42	37	65	91	141	148

Figure 1. HIV Situation in Bangladesh, 2000-2018

A well-planned response to describe the HIV epidemic and estimate the correct number requires accurate information about the disease over time. In such cases, HIV epidemic models are often the most appropriate way to describe the HIV epidemic (Haghdoost, 2011). After the recognition of HIV, the uncertainty about the present and future dimensions of HIV infections led to the development of many models to estimate the current extent of the HIV/AIDS epidemic and to make projections about its future. Some HIV/AIDS model uses reported data which can be used for short-term projections in such areas where case reporting is relatively reliable and complete. But there is a larger possibility of delayed reporting or incomplete reporting, in this circumstance such models need adjustments and the trend will continue in a similar pattern over the next few years which may not give valid projections. In the current global financial climate, reliable estimates are necessary using some valid models to forecast the future trends of HIV cases.

Under these circumstances, this article attempts to estimate the HIV epidemics in Bangladesh as well as obtain the future projections of HIV infection using some analytical models. It is anticipated that the analysis of the findings will help the policymakers and relevant stakeholders to perceive the extent and trends of the HIV pandemic in Bangladesh and to develop country-specific HIV interventions.

2. Materials and Methods

Estimation and Projection Models

For the period 2018-2030, the AIDS Epidemic Model (AEM) and Spectrum model package were used to provide epidemiological and demographic HIV/AIDS estimates and forecasts. Initially, the AIDS Epidemic Model (AEM) was used to estimate HIV prevalence; however, by importing the existing AEM projection into Spectrum, the number of HIV positive people, new HIV infections, AIDS-related deaths, and those in need of ART (anti-retroviral treatment) can be easily estimated and projected.

The AIDS Epidemic Model (AEM) provides potential and realistic pictures of the HIV epidemic and was used mainly for estimating epidemiological and behavioral trends among HIV populations (Brown & Peerapatanapokin, W., 2004). On the other hand, Spectrum has accessibility to more extensive demographic data and can thus assess and predict the demographic, socioeconomic, and other consequences of HIV epidemics (Stover,*et al*., 2006) (Stover, *et al*, 2017). In Spectrum, one of the primary components of which is an AIDS Impact Model (AIM). AIDS Impact Model was used for projecting the consequences of the HIV epidemic, new infections, and AIDS deaths, ART need by age, and sex.

Model Input Assumptions

To produce a Spectrum model, a historical trend of several indicators was required for children and adults, the latter by sex. e. g.

- Adult and adolescent eligibility criteria for ART;
- Number of women getting PMTCT prophylaxis by regimen;
- ANC testing results (for consistency check) Adults receiving ART by sex and children getting ART;
- Children receiving cotrimoxazole and
- HIV status and viral suppression knowledge.

Using this model, the number of new infections can be calculated as (Huq, *et al.*,2020):

$$N_{new\_infection} = n_{contacts\_HIV}\, P_{per\_contact}$$

Where $n_{contacts\_HIV}$ denotes the total number of unprotected contacts with HIV positive partners which can be calculated as:

$$n_{contacts\_HIV} = size\ of\ risk\ group\ \times Frequency\ of\ contacts \\ \times HIV\ prevalence\ of\ partners (\text{Chance to meet positive partners}) \\ \times fraction\ unprotected$$

and,

$$P_{per\_contact} = transmission\ probability\ \times Adjustments\ for\ STI\ \&\ circumcision$$

Moreover, trends by regimen should be provided into the model in the number of women who are pregnant receiving ARVs for prophylaxis. Besides, to determine the impact of HIV, several demographics, epidemiological, and clinical information are optional.

The AEM, on the other hand, has more rigid input requirements than Spectrum. The inputs assume several behavioral and epidemiological indicators as inputs:

- Biological data trends: HIV prevalence, STI prevalence, circumcision in males;
- Behavioral data trends (sexual): Frequency of sexual activity, condom use with different partners, duration of sex work/clienthood (for turnover);
- Behavioral data trends (injecting): Frequency of injection, level of needle sharing, duration;
- Sizes of key populations: Entered as a percentage of adult male or female population;
- Programmatic data trends: Number receiving ART by gender and
- Unit cost data: Unit cost for each program/KP.

Data Source

HIV/AIDS epidemiological data were driven from National Serological and Behavioral Surveillance Reports, Bangladesh Demographic and Health Surveys, and other nationally representative surveys conducted by ICDDR, B, Save the Children, WHO, UNICEF, and the Ministry of Health and Family Welfare's AIDS/STD Program (ASP) to generate historical trends and short-term HIV projections for key indicators (e.g. the number of people living with HIV, the number of new HIV infections, the number of PMTCT, the annual death rate due to AIDS and treatment coverage (ART) for AIDS). These indicators are useful to evaluate the epidemic trends and to observe the overall impact of the national response and in planning for future needs.

Data Limitations

Although most of the data existed, some of the indicators were based on sample size, based on assumptions, e.g. IDU high-risk network and mortality, mobility among key population groups, male circumcision. Additionally, in the case of Key Populations, HIV prevalence numbers for transgender individuals were adjusted because the data is only available for Transgender sex worker people. These modifications have been made with the assistance of specialists from the National HIV Epidemic Estimation Working Group and public health professionals.

3. Results

Epidemiological Estimates and Projections up to 2030

Figure 2 represents the model estimates which reveal that the total number of PLHIV is 13,846 in 2018 and this might reach up to 30,743 by 2030 which is 2.2 times higher than in 2018. The projected number of new HIV-infected population will rise to 2,225 in 2030which is nearly 1.5 times higher compared to the epidemiological estimate in 2018 (1,596). Above all, it can be inferred that the number of HIV-positive people and newly infected people in Bangladesh will continue to grow in the future years

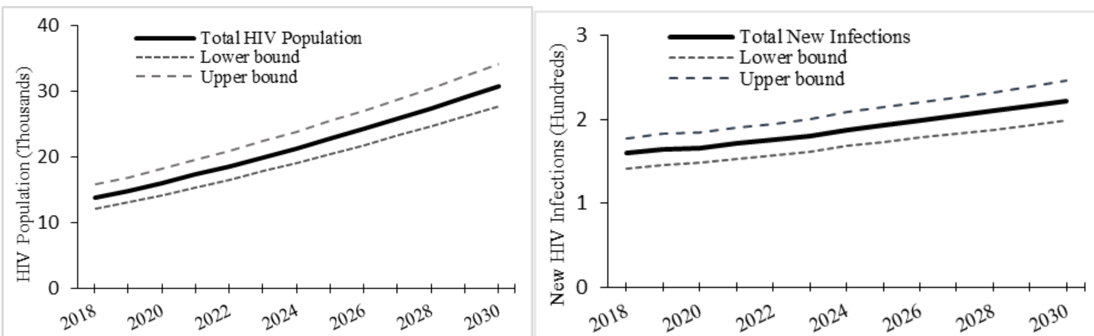


Figure 2. Estimated Number of HIV/AIDS People and New Infections (2018-2030**)**

Percentage of New Infection by Gender Distribution (2018-30)

The projected number of new HIV infections by gender indicates a greater percentage of males (67.5%) than females (32.5%) in 2018. For generalized epidemics, the assumption was that there would be more males infected than females, whereas later in the epidemic there would be more females. The ratio of males to females decreases slowly from 2.07 at the ending of 2018, stabilizing at 1.65 after ten years. The projection of new infections among males represents a downward trend where the projection among females represents an upward trend from 2018 to 2030 (Figure 3).

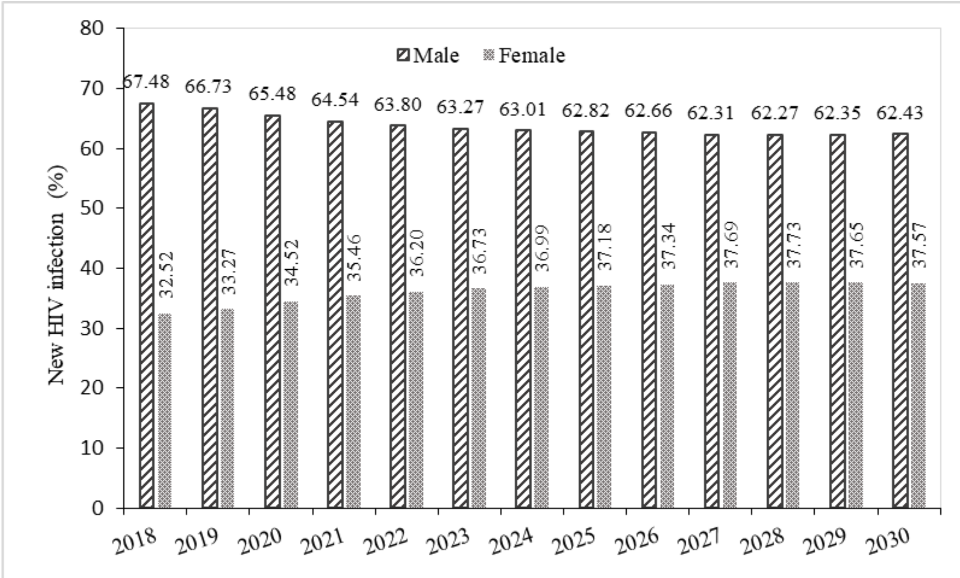


Figure 3. Projection of New HIV Infections by Gender Distribution (2018-2030)

Number of New Infections among Different Age Groups

The different age distribution reveals that there will not be any significant changes in the estimated number of new HIV infections among children (0-14), young (15-24), and adult (25+) population i.e., the number should be slightly changed in the upcoming years. HIV infection is significantly increasing mostly among the adult (25+) population from 2018 to 2030 compared to the young population and children. More specifically, the projection of current infections among adults (25+) will increase up to 1897 in 2030, as compared to 1329 in 2018.

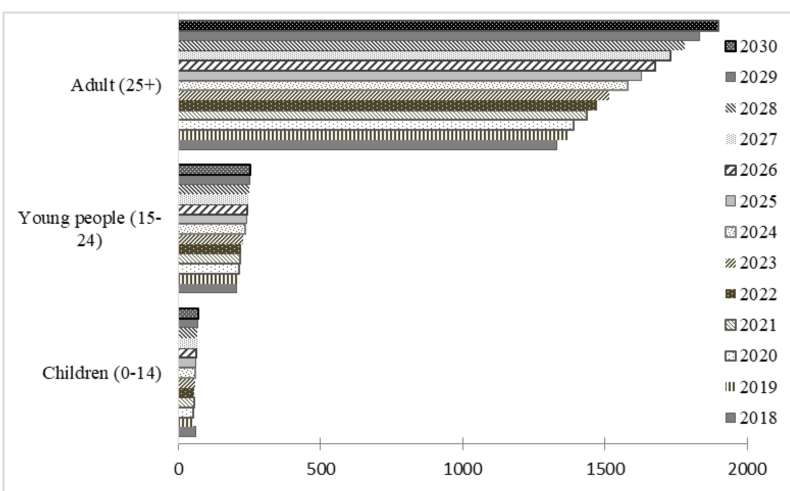


Figure 4. Estimated Number of New Infections among Different Age Groups

HIV Epidemic by Age and Gender Distribution

Table 1 represents the projection of the HIV epidemic for the year 2018-2030 by different age groups and gender. While the number of HIV positive cases, new HIV infections, and ART patients will grow yearly from 2018 through 2030, despite considerable variance in estimates across age groups and gender, annual AIDS deaths will gradually decrease.

The overall number of HIV-infected people is predicted to be13846 with 392 children (0-14) and 13454 adults (15+) in 2018, subsequently, it is increasing and reached a peak in 2030 (527 children, 30215 adults cases). The overall number of new HIV infections is predicted to be 1596 in 2018, with 62 infections in the children (0–14) and 1,534 infections in the adults (15+). The projection till 2030 indicates, the number will increase up to 72 in the children and 2,153 in the adults (15+).

Gender segregation reveals the number of male PLHIV will be increased from 8863 in 2018 to 19329 till 2030, where female PLHIV will be increased from 519 in 2018 to 836 till 2030. The number of new HIV infection cases increased from 1077 cases in 2018 to 1389 in 2030 among males and females, the number increased significantly from 519 in 2018 to 836 in 2030.The estimated annual AIDS deaths among children and adults are 578 in 2018 and the estimated number of deaths will begin to decline slightly, falling to 255 in 2030. The estimated number on ART will be estimated to be 20,969 in 2030 as compared to 2,757 in 2018 which is 8 times higher over the period.

Table 1. Estimated Number of HIV Epidemic in Bangladesh among Different Age Groups and Gender, 2018-2030

Indicators	**Estimated Number of People**							
	Age Distribution	**2018**	**2020**	**2022**	**2024**	**2026**	**2028**	**2030**
PLHIV	Children (0-14)	392	407	423	442	467	499	527
	Adult (15+)	13,454	15,634	18,149	20,866	23,796	26,925	30,215
New HIV Infections	Children (0-14)	62	54	58	61	65	69	72
	Adult (15+)	1,534	1,603	1,696	1,815	1,922	2,030	2,153
Annual AIDS Deaths	Children (0-14)	34	32	33	30	27	29	31
	Adult (15+)	544	301	252	239	215	211	224
Number on ART	Children (0-14)	123	131	139	172	200	211	221
	Adult (15+)	2,634	6,006	8,789	12,349	15,699	18,164	20,749
Indicators	**Gender Distribution**	**2018**	**2020**	**2022**	**2024**	**2026**	**2028**	**2030**
PLHIV	Male	8,863	10,318	11,925	13,608	15,408	17,317	19,329
	Female	4,983	5,722	6,647	7,700	8,855	10,106	11,413

Indicators	**Estimated Number of People**							
	Age Distribution	**2018**	**2020**	**2022**	**2024**	**2026**	**2028**	**2030**
New HIV Infections	Male	1,077	1,085	1,119	1,182	1,245	1,307	1,389
	Female	519	572	635	694	742	792	836
Annual AIDS Deaths	Male	354	183	155	147	128	123	129
	Female	224	150	130	122	114	117	126
Number on ART	Male	1,797	4,226	5,942	8,091	10,093	11,598	13,171
	Female	960	1,912	2,986	4,431	5,806	6,777	7,798

The Number of New HIV Infections by Mode of Transmission among Adults (15+)

Figure 5 reveals that the leading source of HIV infection was needle sharing among the adult population (15+) 574 (37%) in 2018 and it will gradually decrease to 504 (24%) in 2030. The proportion of new infections due to unprotected male-to-male sexual intercourse has climbed from 8% in 2018 to 14% in 2030. Whereas casual sex, the heterosexual relationship among spouses (wife to husband) should remain stable in upcoming years (2018 to 2030). Moreover, in 2018, transmission stands at 384 from husband-to-wife, and sex work at 377, and by the year 2020, these numbers should be 589 and 620 respectively.

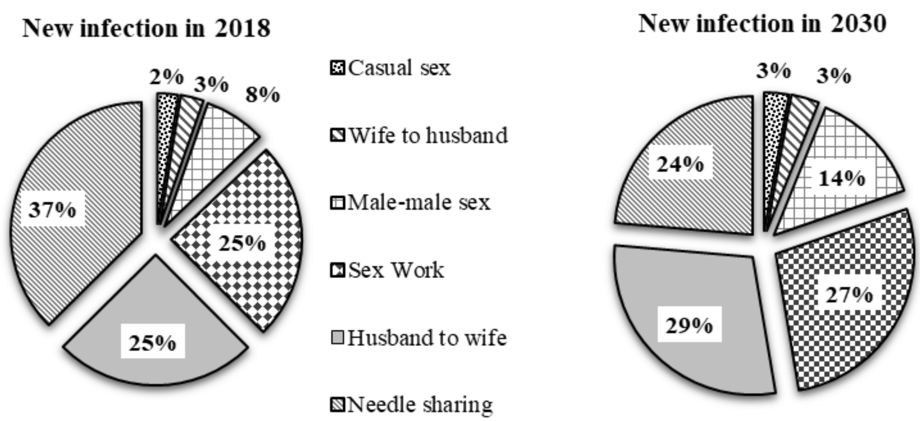


Figure 5. Estimated Number of New Infections by Mode of Transmission among Adults (15+)

People Currently Living with HIV among Adult (15+)

The number of PLHIV among adults (15+) is estimated to be 13,476 in 2018 and might reach 24,564 by 2030, i.e., increasing over the period. (Figure 6). The rapid increase in HIV prevalence among many key population groups indicates that there will be no substantial change in the number of MSW, TG, FSW, and MSM between 2018 and 2030, indicating that the number will remain stable in the coming years. In comparison, between 2018 and 2030, the number of current HIV infections will increase dramatically among PWID and low-risk individuals. In 2030, the number of current HIV infections among low-risk females would increase to 8259, up from 4481 in 2018.

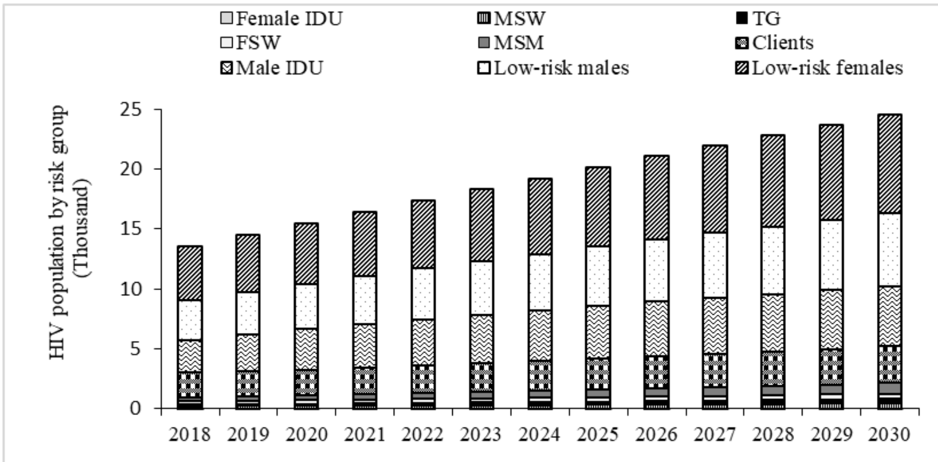


Figure 6. The Estimated Number of People Living with HIV (PLHIV) among Adults (15+)

4. Discussion

The present scenario of the HIV epidemic in Bangladesh is still in its early stages, even though the upward trend of the number of cases reported annually is low but steady. The main focus of this research article was to present the epidemiological profile of HIV infection in the country. More crucially, in the next years, the rising rate of new HIV infections among females will be more severe than among males, needing a greater focus on females. Additionally, HIV transmission occurs mostly through the exchange of discarded needles and syringes between PWID from unprotected paid sex with sex workers and unprotected male to male sex. To address this problem, effective HIV prevention initiatives are urgently needed.

5. Conclusion

Evaluating the scale and patterns of Bangladesh's HIV pandemic, it is anticipated that the analysis of the findings will help the policymakers and relevant stakeholders not only to counteract the deadly consequences of the disease but also to assess the effectiveness of different HIV control or prevention programs that have already been taken in the country. The future projections of the epidemic will also generate evidence to take appropriate measures and design effective prevention programs in Bangladesh.

Limitations

This study utilizes the information up to the year 2018. Situations may have changed by this time which could not be captured by this study due to data limitations.

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Model Selection and Testing Regression Coefficients for Contaminated Data

Imran Hossain Sumon*
Department of Statistics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

Ajit Kumar Majumder
Department of Statistics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

Abstract

Statistical analysis is better when the data are truly representative. However, extraneous stuff such as contamination, irrelevant data, missing observation, distributional faulty in the data cause inaccurate estimation of model parameters. In case of regression analysis, if we remove the extraneous stuff of such data, then we can lose necessary information. On the other hand, traditional estimation and classical test of regression coefficients may fail completely if we retain such type of data. This study aimed to provide an appropriate choice of estimation method, model selection criteria and introducing appropriate testing procedure for regression coefficient in case of contaminated data. Results of this study established that for contaminated and skewed data, robust approach outperforms classical approach to accurate estimation of parameters and identified that outliers are responsible for the inflated residual sum of squares that results in an incorrect solution of model selection and the test of regression coefficients. Findings from this study suggested that we can use the weighted residual sum of squares instead of the residual sum of squares to avoid such an inflated problem.

***Key Words**:* Outliers; Skewness; Robust regression; Model selection; Testing Coefficients

1. Introduction

Regression analysis is the most widely used statistical technique for fitting models to real-life data and is regularly applied to most sciences. Regression analysis goes forward with several basic steps including model building, model selection, and hypothesis testing. The hypothesis test ensures whether underlying assumptions and methods of estimation are suitable for data or not. The fitted model is used to predict the future behavior of variables. However, before going to forecast by a model, we have to justify whether this model is adequate or not. In regression analysis, one of the most important tasks is to control extreme observations (outliers) because they are sensible for certain types of models. For example, in the presence of outliers and skewed data, the ordinary least square (OLS) is unable to produce robust estimates. To fix this problem, two methods are suggested in the literature (Rousseeuw & Leroy, 1987). The first one is regression diagnostics (Rousseeuw & Leroy, 1987), however, in the presence of multiple outliers, it is unfortunately much more difficult to diagnose them. The other approach is robust regression (Rousseeuw & Leroy, 1987). Because in the case of outliers and contaminated data, some of the OLS assumptions (e. g., error follows the normal distribution with constant variance) are violated, and for that reason, OLS does not provide appropriate result. In OLS, we

* Corresponding author: imranhossainsumon38@gmail.com

minimize the residual sum of squares that is extremely affected by outliers and contaminated data. Moreover, previous studies pointed out that in the presence outlier, weighted (robust) mean (Hossain, 2016) and variance (Hossain, 2017) provide more accurate estimates than their classical version. For that reason, we should use the robust regression method such as LAD, LMS, and M-estimation to obtain better results in such situations.

Model selection is another important part of regression analysis. In this procedure, firstly, we must fit several models and then among them, we select the most appropriate model by using a numerical summary of their goodness-of-fit, properties or combination of both for prediction (Määttä et al., 2016). Sequential testing, allowing variables to be added or deleted at each step, has often been employed. However, such p-value-based testing techniques only evaluated two nested models and have been widely criticized, as hypothesis tests are a poor basis for model selection in general. Cross-validation and its variation have been suggested and discussed as a useful model selection method (Akaike, 1974). The adjusted coefficient and Mallow's C_p statistic is also widely used in classical regression analysis and provide a ranking for all considerable models. Moreover, several model selection criterion, such as R^2 Criterion, Root mean square deviation $(RMSD)$, Adjusted R^2, Criterion Akaike information criterion (AIC), Akaike information criterion with small sample correction $(AICc)$, Bayesian information criterion (BIC), Schwartz's information criterion (SIC), Hannan-Quinn information criterion (HQC), Mean absolute error (MAE) and their variations will also be considered (Draper & Smith, 1981).

The alternative to OLS regression in the case of outliers is robust regression. Robust regression modeling has been applied in several studies. The introduction of Least Median of Squares approach comes from Rousseeuw (1984). In order to obtain LAD estimators, Charnes et al. (1954) addressed linear programming. Portnoy & Koenker (1997) provide a comprehensive summary of the LAD approach. Koenker & Bassett (1978) and Pollard (1991) obtained large sample properties for the LAD estimates. Alma (2011) concluded that the S-estimator would increase its effectiveness in case of 10% breakdown (Almetwally & Almongy, 2018). MM-estimation works best in contrast to a wide set of extrinsic condition (Almetwally & Almongy, 2018). From the descriptive point of view, we can say that the criteria and usual hypothesis testing for ordinary square regression may fail miserably even for a large sample. Khan & Majumder, (2012) introduced weighted model selection criteria to identify the best-fitted model under the OLS, LAD, and LMS regression in case of contaminated data. However, to the author's knowledge, no scholarly article has been developed on hypothesis testing in the case of the contaminated and skewed data under these estimation approaches. Therefore, this study attempts to fulfill this methodological gap. This research aims to compare the performance of OLS, LAD, and LMS methods and test the regression coefficients by using the proposed WRSS based test statistics in case of contaminated data.

2. Methods and Materials

Most well-known regression estimator is the ordinary least squares (OLS) estimator, $\hat{\beta}_{LS}$ which is defined as the vector that minimizes residual sum of squares (Gujarati, 2004).

$$SSE = \sum_{i=1}^{n}\left(y_i - \hat{y}_i \right)^2 \tag{2.1}$$

But the major shortcoming of the traditional approaches to linear regression has always been the fact that in the regression analysis one assumes that the errors are truly random or accidental. Blunders, clerical errors, misprints in published or electronically stored data are simply ignored in the analysis (Giloni & Padberg, 2002). "Whereas", this type of data has a large influence on the classical approach. The classical approach is also highly influenced by outliers and contaminated data. Sometimes outliers are the result of a systematic problem with either our data collection techniques or our model (McCann, 2005). The outliers occurring with extreme values of the regressor variables can be especially disruptive (Thanoon, 2015). Contamination can, of course, take many forms. It may be recording or reading errors. In this case, correction or rejection might be the only possibility. Alternatively, it might reflect low incidence mixing of x with another random variable whose source and manifesto are uninteresting. For this problem, McCann (2005) proposed a well-earned approach for dealing with outliers and contamination. These are the robust (or resistant) methods such as least median square (LMS), least absolute deviation (LAD), least trimmed squares (LTS), M-estimation, S-estimation etc.

LAD represents the method of solving an over-defined system of (linear) algebraic equations mentioned by Taylor (1974) is related to KF Gauss and PS Laplace. Mathematicians suggest to minimizes the sum of absolute residuals in the equations (Dasgupta & Mishra, 2011).

$$\underset{\hat{\beta}}{minimize} \sum_{i=1}^{n} \left| \varepsilon_i \right| \tag{2.2}$$

This is also called L_1 regression (or L_1 - norm regression) whereas least square is L_2 (Draper & Smith, 1981). The least squares regression is very far from the optimal in many non-Gaussian situations, especially when the errors follow distributions with longer tails (Thanoon, 2015). Unlike the *LS* method, the *LAD* method is not sensitive to outliers and produces robust estimates (Chen et al., 2008).

A different approach, least median squares *(LMS)* is introduced which minimizes the median of the squared residuals (Rousseeuw, 1984). That is replacing "sum" in OLS by median yield the *LMS* estimator of the parameter.

The least median of squares *(LMS)* estimator minimizes the objective function,

$$\underset{\hat{\beta}}{minimize}\ \underset{i}{med}\left(y_i-\hat{y}_i\right)^2=\underset{\hat{\beta}}{minimize}\ \underset{i}{med}\ \varepsilon_i^{\ 2} \tag{2.3}$$

ere, $\varepsilon_i^{\ 2}\left(i=1,2,\ldots,n\right)$ are the residual squares. The solution is that β that produces the minimum such median (Draper & Smith, 1981). In the case of least squares, the notion of the breakdown point $\grave{o}^*$ is zero. That is,

$$\grave{o}^*=0$$

But whereas the breakdown point of the univariate median is as high as 50% (Hampel, 1971).

2.1 Weighted Model Selection Criteria

We know that the residual sum of squares is used in all the classical model selection criteria though RSS is very much affected by outlier. To this problem, we have to use the weighted residual sum of squares s (WRSS) instead of RSS, which is given by

$$WRSS=\sum_{i=1}^{n}w_i(y_i-\hat{y}_i)^2 \tag{2.4}$$

Where,

w_i is the weight for i^{th} observation

y_i denotes the true value for i^{th} trial of the regressand variable,

$\hat{y}_i$ is the predicted value for i^{th} trial of the regressand variable.

Now w_i can be computed as

$$w_i=\begin{cases}1 & if\left|\dfrac{\varepsilon_i}{s^0}\right|<2.5\\ 0 & otherwise\end{cases}$$

(Rousseeuw, 1987, pp-44)

And the scale estimate is denoted by s^0 is defined as

$$s^0=1.4826\left(1+\frac{5}{n-p}\right)\sqrt{\underset{i}{med}\ \varepsilon_i^2}$$

(Rousseeuw, 1987, pp-44)

We propose to use the weighted residual sum of squares (WRSS) in place of residual sum of squares (RSS) in all the classical model selection criteria to obtain better fitted model in case of contaminated data (Khan & Majumder, 2012).

2.2 Proposed Test for Overall Regression Coefficients

In regression analysis firstly performing test is often a test of whether the regressor variables have any significant effect on the dependent variable. Let us consider the general form of the linear regression model

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p + \varepsilon \tag{2.5}$$

Suppose we want to test the following hypothesis

$H_0 : \beta_1 = \beta_2 = \ldots = \beta_p$

$H_1 :$ At least one β_j 's is not equal to zero, $j = 1, 2, \ldots, p$

Under the null hypothesis the reduced model becomes

$$y = \beta_0 + \varepsilon^*$$

Now the proposed test statistics is,

$$F = \frac{\left(WSSR_{reduced} - WSSR_{full}\right) / (p-1)}{WSSR_{full} / (n-p)} \tag{2.6}$$

Assuming the normality of the distribution of random errors F has an F distribution with numerator and de-numerator degrees of freedom $p-1$ and $n-p$ respectively. Where, k is the number of explanatory variables including intercept.

At $\alpha\%$ level of significance the critical value of F with numerator and de-numerator degrees of freedom $p-1$ and $n-p$ respectively is $F_c = F_{p-1, n-p}(\alpha)$. If the calculated value of F is greater than the critical value of F, then we may reject the null hypothesis. That is, there are significant effects of the explanatory variables on the dependent variable. Otherwise, we may not reject the null hypothesis.

2.3 Proposed Test for Individual Regression Coefficient

Let us consider the general linear regression of equation (2.5). Consider that the weighted residual-based F test provides information that there are significant effects of the explanatory variables on the dependent variable (i. e., null rejected). Then we eager to specify the explanatory variable for which there is a significant effect on the dependent variable.
Now let us consider the following hypothesis

$$H_0 : \beta_j = 0 \qquad \text{against}$$
$$H_1 : \beta_j > 0\,, \; j = 1, 2, \ldots, p$$

The test statistics under the null hypothesis is given by

$$\left|t_j\right| = \frac{\left|\hat{\beta}_j\right|}{SD\left(\hat{\beta}_j\right)} \tag{2.7}$$

To obtain the test statistics, we have to calculate an estimate of the standard deviation of $\hat{\beta}_j$ by substituting $\hat{\sigma}$ for σ by using the following formula,

$$SD\left(\hat{\beta}_j\right) = \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^{n}\left(x_{ij} - \overline{x}_{.j}\right)^2}}$$

Where,

$$\hat{\sigma} = \sqrt{\frac{WSSR}{n - p - 1}}$$

x_{ij} is the value of the i^{th} observation for j^{th} explanatory variable

$\overline{x}_{.j}$ is the mean value of j^{th} explanatory variable

If we specify the shape to be normal (bell-shaped), that is, if we assume the normal linear regression model, then the resulting distribution of the test statistics t is still close to the t distribution with $n-2$ degrees of freedom. Assuming the normality of the error term, the critical value of t at $\alpha\%$ level of significance is $t_c = t_{(n-2)}\left(\alpha/2\right)$. If the calculated value of t_j is greater than the t_c, then we may reject the null hypothesis. That is, there is significant effect of the explanatory variable x_j on the dependent variable. Otherwise, we may not reject the null hypothesis.

2.4 Data Source

This paper uses GNP data from Table B-1, p. 232; third measure money stock data from Table B-61, p. 303 of the economic report of the president (1985) for scrutinizing the performance of different estimations methods in simple regression by proposed model selection criteria. The explanatory variable represents the third measure of money stock data, and the response variable is gross national product (GNP). The data was also used by Gujarati (2004), and others.

3. Results

Consider four different case to make comparison of the methods OLS, LMS and LAD. In first case, the comparison is made in simple regression for mentioned data without outlier. Afterward a single outlier is taken in the response variable Gross national product ($i.e.$, in y direction) and presented in this section as second case. Single outlier is taken in the explanatory variable money stock measure ($i.e.$, in x direction) and presented in this section as third case. Outliers are taken in both the dependent variable ($i.e.$, in y direction) and independent variable ($i.e.$, in x direction) in the fourth case. The results obtained in each case are presented below:

Uncontaminated Data

The classical regression estimation procedure ($i.e.$, OLS) is as usual. On the other hand, the objective functions of LMS and LAD methods are not straightforward that's why the estimation is not unique. Thus, an iterative procedure is adopted in estimating the fitted models for both methods. The fitted models by the method of OLS, LAD and LMS are respectively given in the next:

$$\hat{y}_{OLS} = 158.788 \ + 1.204X \tag{3.1}$$

$$\hat{y}_{LAD} = 140.968 + 1.207X \tag{3.2}$$

$$\hat{y}_{LMS} = 169.737 + 1.181X \tag{3.3}$$

From the figure 1, we observe that the OLS, LAD, and LMS methods give almost the same fitted line for the uncontaminated data. Also, classical model selection criteria (see Table 1 in appendix) show that all three methods perform very closely in order to fit the approximate model. Although the result of the model selection criterion is almost same, but all the criteria suggest that OLS is better than other methods in case of uncontaminated data. Weighted model selection criteria (see Table 1 in Appendix) shows that LAD is better than other methods. Classical F-test (see Table 1 in Appendix) and proposed test (see Table 1 in Appendix) indicate that linear regression model equation 3.1, 3.2, 3.3 provides a better fit to the data than a model that contains no independent variables. Both the classical t-test (see Table 1 in Appendix) and proposed t-test (see Table 1 in Appendix) indicate that money stock data have a significant association with GNP. Whereas, intercept coefficient is also significant

Contamination in y-Direction

In this section, a single outlier is taken in GNP of the same data used in immediate previous case. The fitted models by the method of OLS, LAD and LMS are respectively given in the next:

$$\hat{y}_{OLS} = -7585.5 + 7.718X \tag{3.4}$$

$$\hat{y}_{LAD} = 40.0467 + 1.302X \tag{3.5}$$

$$\hat{y}_{LMS} = 169.737 + 1.181X \tag{3.6}$$

All the fitted models from equation 3.1 to equation 3.3 indicate that the intercept is positive. But if we consider outlier in the y-direction of the data then OLS fitted model shows a negative intercept (see equation 3.4), which designates a failure of the OLS method to estimate the actual model. Whereas, the LMS and LAD fitted models computed the actual signs of the relationship.

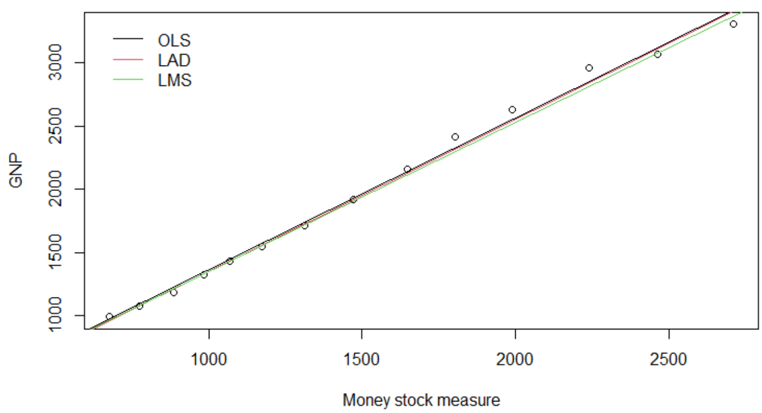

Figure 1. The Observed and Fitted y against Observed x for Uncontaminated Data

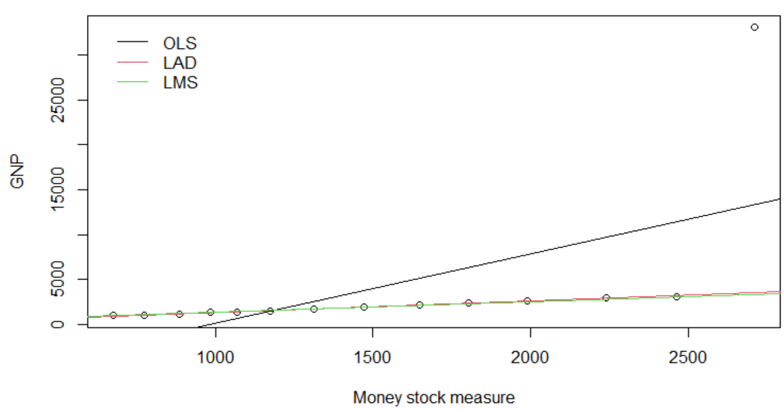

Figure 2. The Observed and Fitted y against Observed x for Contaminated Data in y-Direction

Graphical presentation of the fitted GNP against Money stock measure ($i.e.$, presented in equation 3.4 through equation 3.6) is exhibited in the Figure 2. Figure 2 reflects that the LMS and LAD methods yield a good fit for contaminated data in the y-direction. On the other hand, the OLS method has been affected by the outlier data, and therefore the intercept is so much different from the original one (see Figure 1). But all the classical model selection criteria (see Table 2 in Appendix) indicate that the estimated OLS model is better than the estimated LAD and LMS models, which is contradictory to the graphical presentation. On the other hand, examining all the weighted model selection criteria (see Table 2 in appendix) reveals that the fitted LMS and LAD models are better than the fitted OLS model which is exactly similar to the graphical representation and the actual relationship that exist between GNP and money stock measure. It is also revealed that LAD method is better than LMS method. Classical F-test (see Table 2 in Appendix) unveils that fitted OLS line (see equation 4.4) is better than a model that contains no independent variables whereas the conclusion is different under LAD and LMS approaches. The proposed weighted F test (see Table 2 in Appendix) reflects that fitted LAD and LMS provide a better fit than a model with the only intercept but not for the fitted OLS line. The classical t-test (see Table 2 in appendix) indicates that there is no significant association between money stock measure and GNP under the LAD and LMS methods but there is a significant association between money stock measure and GNP under the OLS method whereas the intercept coefficient is not significant for all the method. The proposed weighted t-test (see Table 2 in appendix) unveils that there is a significant association between money stock measure and GNP under all methods whereas the intercept coefficient is significant for LAD and LMS methods but not for the OLS method.

Contamination in x-direction

In this section, a single outlier is taken in money stock measure of the same data used in uncontaminated data. The fitted models by the method of OLS, LAD and LMS are respectively given in the next:

$$\hat{y}_{OLS} = 1777.10 + 0.0630X \tag{3.7}$$

$$\hat{y}_{LAD} = 1637.28 + 0.0615X \tag{3.8}$$

$$\hat{y}_{LMS} = 169.737 + 1.181X \tag{3.9}$$

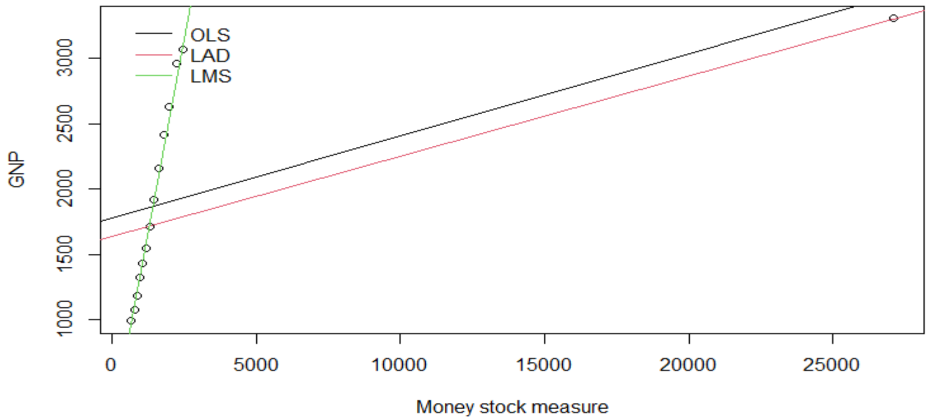

Figure 3. The Observed and Fitted y against Observed x for Contaminated Data in x-Direction

Figure 3 shows that the LMS method yield a good fit than other two methods for contaminated data in the x-direction. Fitted models (see equation 3.7 and 3.8) for the contaminated data in x-direction under OLS and LAD methods are different from the models (see equation 3.1 and 3.2) for original data whereas the fitted model (see equation 3.9) for the contaminated data in x-direction under LMS methods are different from the model (see equation 3.3) for original data. It is evident from the classical model selection criteria (see Table 3 in Appendix) that OLS method is better compare to LAD and LMS methods for the contaminated data in X -direction which is aberrant to the graphical presentation. That signify the classical model selection criteria fail to identify the best method that fit to the majority of the data. According to all the weighted model selection criteria (see Table 3 in Appendix) LMS method is better than the OLS and LAD methods. Classical F-test unveils that the fitted model under only the OLS method is better than the model with the only intercept coefficient. According to the proposed weighted F-test fitted model under the all the methods is better than the model with the only intercept coefficient. The classical t-test (see Table 3 in appendix) indicates that there is no significant association between money stock measure and GNP under the LAD and LMS methods but there is a significant association between money stock measure and GNP under the OLS method whereas the intercept coefficient is not significant for LMS method. The proposed weighted t-test (see Table 3 in appendix) unveils that there is a significant association between money stock measure and GNP under all methods whereas the intercept coefficient is also significant for all methods.

Contaminated Data both in X-Direction and y-Direction

The fitted models by OLS, LMS and LAD method after considering the outliers in both the x-direction and y- direction are as like as:

$$\hat{y}_{OLS} = 2937.88 + 0.2040X \tag{3.10}$$

$$\hat{y}_{LAD} = 2090.22 + 0.0447X \tag{3.11}$$

$$\hat{y}_{LMS} = 31.933 + 1.302X \tag{3.12}$$

Figure 4 reflects that LMS provide better fit than LAD and OLS. By examining all the classical model selection criteria (see Table 4 in Appendix), it may conclude that the OLS method is better than the LMS and LAD method which is aberrant to the graphical presentation. It signifies that the classical model selection criteria fail to identify the best-fitted model for contaminated data in both directions. On the other hand, all the weighted model selection criteria (see Table 4 in appendix) indicate that the LMS fitted line are better than the OLS and LAD fitted lines which is exactly similar to the graphical representation. Classical F-test (see Table 4 in appendix) unveils that the model with the only intercept coefficient is better than the fitted model under the OLS, LAD, and LMS methods. According to the proposed weighted F-test (see Table 4 in appendix), the fitted model under the OLS and LMS methods is better than the model with the only intercept coefficient. The proposed weighted t-test (see Table 4 in appendix) unveils that there is a significant association between money stock measure and GNP under OLS and LMS methods whereas the intercept coefficient is significant for all methods.

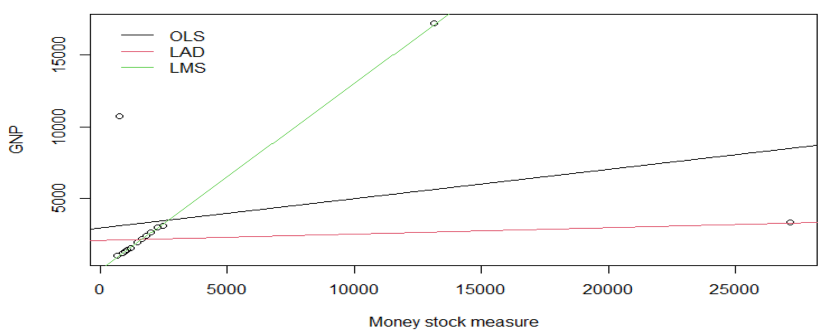

Figure 4. The Observed and Fitted y against Observed x for Contaminated Data in both x-Direction and y-Direction

5. Conclusion

This paper develops weighted test statistics in order to test the regression coefficients and applied the existing weighted model selection criteria for the purpose of comparison of the different estimation methods of the linear regression and it also develops weighted test statistics in order to test the regression coefficients. Analysis tells us, the weighted model selection criteria based on WRSS perform well for contaminated data, while the usual model selection criteria fail to identify the best method in fitting the regression model. Another important fact is that weighted model selection criteria fail to identify the best-fitted regression model in the case of uncontaminated data. According to weighted model selection criteria, the LMS method gives a better fit than other two methods for contaminated data in x-direction and y-direction. It is also palpable from simple regression for contaminated data both in X-direction and y-direction that the LMS method is the best among the three compared methods. It is evident from the analysis part is that the weighted test statistics based on WRSS perform well for contaminated data, while the usual test statistics fail to test regression coefficients.

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Appendix

Table 1. Classical and Weighted model selection criteria of simple linear regression for uncontaminated data

Criterion	Classical	Weighted	Classical	Weighted	Classical	Weighted
	OLS		LAD		LMS	
RMSD	56.816	13.815	58.341	7.1154	62.915	11.262
AIC	4295.6	253.969	4529.3	67.373	5267.32	168.787
AICc	4296.6	255.059	4530.4	68.464	5268.41	169.878
SIC	4706.2	278.246	4962.3	73.813	5770.83	184.921
R^2	0.9943	0.9997	0.9940	0.9999	0.9930	0.9997
Adjusted R^2	0.9938	0.9996	0.9935	0.9999	0.9924	0.9997
MAE	44.417	10.240	40.533	3.108	41.619	6.8023
FPE	4304.0	234.894	4538.3	67.505	4871.72	169.119
Hypothesis testing by using classical and weighted test F statistics						
F **-statistics (p-value)**	2099.17 (<0.001)	22632.8 (<0.001)	2007.72 (<0.001)	19266.5 (<0.001)	2900.07 (<0.001)	27860.6 (<0.001)
P value of the corresponding coefficient by using classical and weighted test t statistics						
p_0	0.0031	(<0.001	0.0132	(<0.001	0.0014	(<0.001
p_1	<0.001	(<0.001	(<0.001	(<0.001	(<0.001	(<0.001

Table 2. Classical and Weighted model selection criteria of simple linear regression for contaminated data in y-direction

Criterion	Classical	Weighted	Classical	Weighted	Classical	Weighted
	OLS		LAD		LMS	
RMSD	6450.89	203.34	7876.62	54.42	7929.82	60.37
AIC	55376364	55022	82558825	3941.5	83677806	4850.7
AICc	55376365	55023	82558826	3942.6	83677807	4851.8
SIC	60669844	60282	90450702	4318.4	91676648	5314.3
R^2	0.35816	0.9994	0.0431	1	0.0301	0.9999

Adjusted R^2	0.3047	0.9993	0.0000	1	0.0000	0.9999
MAE	4257.216	59.247	2135.66	30.59	2156.16	36.89
FPE	55485440	50890	82721443	3949.4	77393195	4860.2
Hypothesis testing by using classical and weighted test F statistics						
***F* -statistics (p-value)**	6.6961 (0.0238)	1.3199 (0.2730)	1.4719 (0.2483)	14794 (<0.001)	1.8166 (0.2026)	27861 (<0.001)
P value of the corresponding coefficient by using classical and weighted test t statistics						
p_0	0.1466	0.2403	0.9893	(<0.001	0.7867	(<0.001
p_1	0.0238	(<0.001	0.4763	(<0.001	0.5442	(<0.001

Table 3. Classical and Weighted model selection criteria of simple linear regression for contaminated data in x-direction

Criterion	**Classical**	**Weighted**	**Classical**	**Weighted**	**Classical**	**Weighted**
	OLS		**LAD**		**LMS**	
RMSD	234.594	127.036	643.443	97.938	7716.98	60.375
AIC	57280.1	21475.1	550939.3	12764.1	79246201	4850.7
AICc	57280.2	21476.2	550940.4	12765.2	79246202	4851.8
SIC	60343.7	23527.9	603604.2	13984.2	86821421	5314.3
R^2	0.3078	0.9716	0.2709	0.9831	0.0000	0.9935
Adjusted R^2	0.3007	0.9692	0.2102	0.9817	0.0000	0.9930
MAE	75.33	68.207	515.77	44.262	2099.276	36.891
FPE	57280.4	19862.2	552024.5	12789.2	73294425	4860.2
Hypothesis testing by using classical and weighted test F statistics						
***F* -statistics (p-value)**	5.336 (0.0395)	10.664 (0.0068)	4.604 (0.0530)	14794 (<0.001)	4.734 (0.0513)	27861 (<0.001)
P value of the corresponding coefficient by using classical and weighted test t statistics						
p_0	(<0.001	(<0.001	(<0.001	(<0.001	0.0674	(<0.001
p_1	0.0395	0.0215	0.1387	0.0078	0.1534	(<0.001

Table 4. Classical and Weighted model selection criteria of simple linear regression for contaminated data in both x and y direction

Criterion	**Classical**	**Weighted**	**Classical**	**Weighted**	**Classical**	**Weighted**
	OLS		**LAD**		**LMS**	
RMSD	1549.92	1241.39	617.00	97.938	8952.34	56.233
AIC	2500288	2050707	506593	12764.1	106649118	4207.91
AICc	2500289	2050708	506594	12765.2	106649118	4209.0
SIC	2634016	2246736	555019	13984.2	116843808	4610.15
R^2	0.1087	0.9199	0.9802	0.9831	0.0002	0.9998
Adjusted R^2	0.0993	0.9132	0.9786	0.9817	0.0000	0.9998
MAE	383.73	1001.15	482.19	44.262	3014.235	33.063
FPE	2500302	1896689	507590	12789.2	98639249	4216.20
Hypothesis testing by using classical and weighted test F statistics						
F **-statistics (p-value)**	1.4585 (0.2504)	11.052 (0.0614)	0.2353 (0.6364)	6.7251 (<0.2357)	0. 3961 (0.0513)	7482.53 (<0.001)
P value of the corresponding coefficient by using classical and weighted test t statistics						
p_0	0.055	(<0.001	0.1125	(<0.001	0.1867	(<0.001
p_1	0.250	(<0.001	0.7810	0.1007	0.8442	(<0.001